# Intonation as a quantifier-free logical interpretation of metrical and prosodic structure

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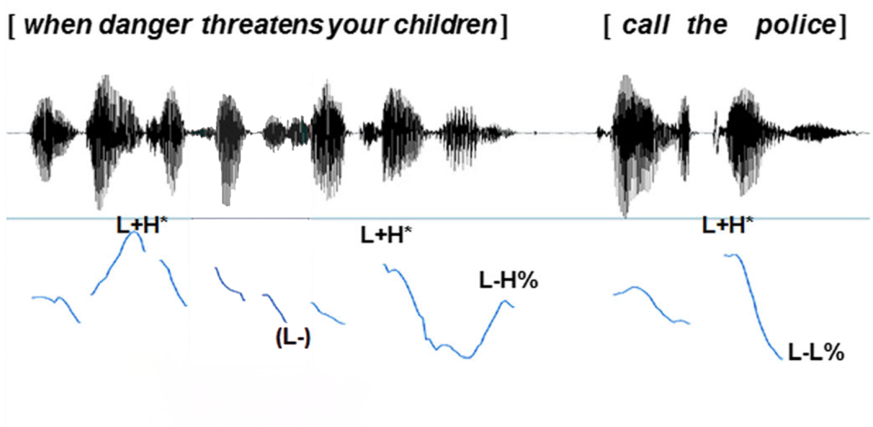
## 1 Introduction

- How can we analyze *intonation*?




- Just like lexical tones (e.g., Goldsmith, 1976), intonation can be autosegmentally represented with a sequence of Highs (Hs) and Lows (Ls) in Autosegmental-metrical (AM) theory of intonation (e.g., Pierrehumbert, 1980; Ladd, 2008).

- We can represent intonation as in (1).

(1) [wɛn deɪn dʒə θɹe tnz jʊə tʃɪl dɹən]

$\rtimes_\iota \rtimes_\varphi \quad \sigma \quad \sigma^* \quad \sigma \quad \sigma \quad \sigma \quad \sigma \quad \sigma^* \quad \sigma \quad \ltimes_\varphi \ltimes_\iota$

$\rtimes_\iota \rtimes_\varphi \quad$ L+H* $\quad$ L- $\quad$ L+H* $\quad$ L- $\quad$ H% $\quad \ltimes_\varphi \ltimes_\iota$

- But importantly, unlike lexical tones that are associated to every TBUs, intonational tones seem to be associated with TBUs non-locally due to their associations with particular TBUs that are in *metrically strong positions*, such as starred syllables or phrasal-final syllables.

- Jardine (2017) analyzed the autosegmental representation of lexical tones (e.g., Goldsmith, 1976) using a *logical interpretation* (Courcelle, 1994) of the input structure of toned syllables.

- Questions:

  1. How can we define the autosegmental representation of intonation as a logical interpretation and how does this say about the computational nature of intonation?

  2. How can we posit a computational theory of intonation that predicts a typology of possible intonational patterns of languages based on the computational nature?

The present study:

- Here we view *intonation* as a quantifier-free (QF) logical interpretation of a metrical and prosodic structure.
- Importantly, tones in intonational melodies are literal copies of elements in the metrical and prosodic structure, such as starred syllables or boundaries.
- Results show that head- and edge-prominence intonational patterns are QF interpretations of metrical grids, whereas lexical pitch accent patterns are more complex.

## 2 Previous studies

### 2.1 Empirical domain: intonation

- According to the AM theory, intonational structure is hierarchically organized with pitch accent, phrasal tone, and boundary tone.
- Importantly, these tones are associated with *metrically strong positions* in an utterance (e.g., a head of a constituent, phrasal boundaries).

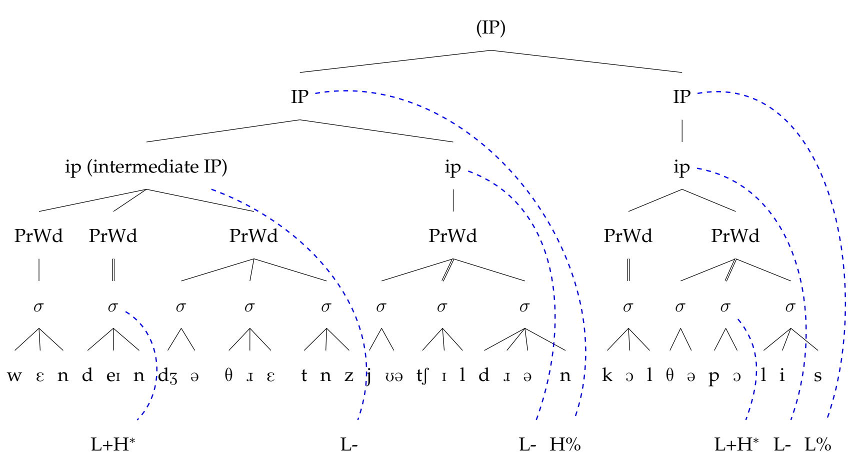



Figure 1: A prosodic structure of American English for an utterance *When danger threatens your children, call the police*. Redrawn from Cho (2016).

- Jun (2005) provided the prosodic typology depending on the prominent and rhythmic/prosodic elements, focusing on *describing* the intonational patterns.

| | **Prominence** | | | | | **Rhythmic/prosodic unit** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Language** | lexical | | | post-lexical | | lexical | | | post-lexical | | |
| | tone | stress | LPA | head | edge | mora | syll | foot | AP | ip | IP |
| English | | x | | x | | | | x | x | | x |
| Japanese | | | x | x | x | x | | | x | (x) | x |
| Korean | | | | | x | | x | | x | | x |

Table 1: Prosodic typology (Jun, 2005). Lexical pitch accent (LPC), Accentual Phrase (AP).

- I am wondering:

  (1) What are the computational properties of TBUs and intonational tones?

  (2) How can we define the association relationship between TBUs and intonational tones?

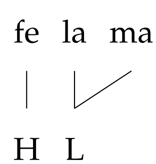
  How can we make a typological predictions on (1) and (2)?

- Here I will use the logical interpretation of intonational patterns to make this typological information more *explicit*.
- I will do some case studies of tone-TBU associations in intonation, by looking at three different intonation patterns: a *head-prominence* language, American English; an *edge-prominence* language, Seoul Korean; a *lexical pitch accent* language, Tokyo Japanese.

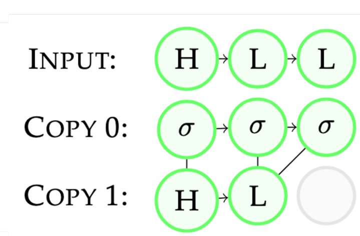
## 2.2 Logical interpretations of autosegmental representations

- Jardine (2017) studied that autosegmental representation is an *interpretation* of linear representation, using *logical interpretation* (Courcelle, 1994; Engelfriet & Hoogeboom, 2001; Filiot & Reynier, 2016).

(2) a. félàmà 'junction' b.




| | |
|---|---|
| COPY 0 | $\sigma^0_o(x) = \texttt{True}$ |
| COPY 1 | $\mathrm{H}^1_o(x) = \mathrm{H}_i(x) \wedge \texttt{spanfirst}(x)$<br>$\mathrm{L}^1_o(x) = \mathrm{L}_i(x) \wedge \texttt{spanfirst}(x)$ |
| ASSOCIATION | $\mathcal{A}^{0,1}_o(x,y) = \texttt{spanfirst}(y) \wedge (x \approx y \vee \texttt{span}(y,x))$ |




- Following Jardine (2017), we are viewing *intonation* as a logical interpretation of an input structure.

# 3 Model theory and logic

(3) $\langle\ D = \{0, 1, 2, 3, 4, 5\};$

$P_{\text{C}} = \{1,3\},$

$P_{\text{V}} = \{2,4\},$

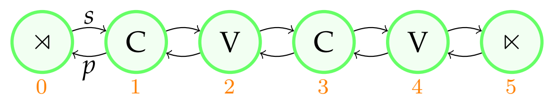
$P_{\rtimes} = \{0\},$

$P_{\ltimes} = \{5\},$

$p = \{(0,1), (1,2), (2,3), (3,4), (4,5)\},$

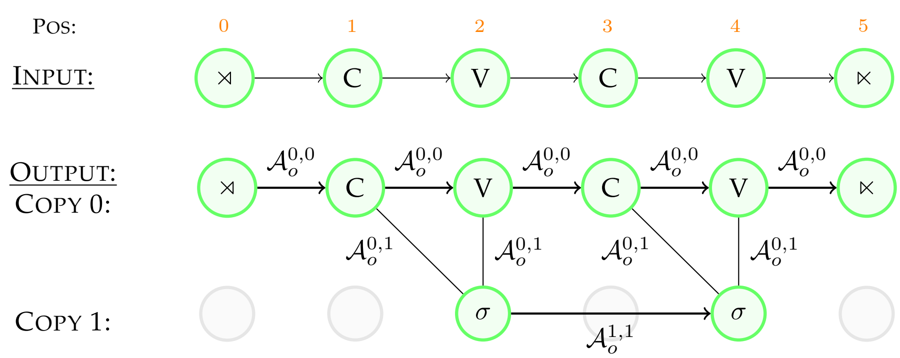
$s = \{(1,0), (2,1), (3,2), (4,3), (5,4)\}\ \rangle$

(4)




- Following Strother-Garcia (2017), we view the CVs in the string that is *interpretable* as syllables.
- Importantly, we define the semantics of $\mathcal{S}_{\text{o}}$ as *order-preserving* following Chandlee & Jardine (2019).

| | |
|---|---|
| COPY 0 | $\text{C}_{\text{o}}{}^{0}(x) = \text{C}_{\text{i}}(x)$<br>$\text{V}_{\text{o}}{}^{0}(x) = \text{V}_{\text{i}}(x)$<br>$\rtimes_{\text{o}}{}^{0}(x) = \rtimes_{\text{i}}(x)$<br>$\ltimes_{\text{o}}{}^{0}(x) = \ltimes_{\text{i}}(x)$ |
| COPY 1 | $\sigma_{\text{o}}{}^{1}(x) = \text{V}_{\text{i}}(x)$ |
| ASSOCIATION | $\mathcal{A}_{\text{o}}{}^{0,1}(x, y) = (\text{C}_{\text{i}}(x)\ \wedge \text{V}_{\text{i}}(y)\ \wedge\ y \approx s(x)) \vee (\text{V}_{\text{i}}(x)\ \wedge\ \text{V}_{\text{i}}(y)\ \wedge\ y \approx x)$ |



## 4 Intonation as quantifier-free interpretation

- We now define intonation using a logical interpretation, where the output structure is defined based on the input structure.

(5) $\mathcal{S}_{\text{i}} = \{\sigma, \sigma^*, \ltimes_\varphi, \ltimes_\iota, \rtimes_\varphi, \rtimes_\iota, p, s, p^*, s^*\}$

(6) $\mathcal{S}_{\text{o}} = \{\sigma, \sigma^*, \text{T}, \text{T}^*, \ltimes_\varphi, \ltimes_\iota, \rtimes_\varphi, \rtimes_\iota, \mathcal{A}, p, s, p^*, s^*\}$

  - $\sigma, \sigma^*$: Tone Bearing Units (TBUs)
  - $\rtimes_\varphi, \ltimes_\varphi$: ip boundaries
  - $\rtimes_\iota, \ltimes_\iota$: IP boundaries
  - T, T*: Tones, pitch accented tones
  - $\mathcal{A}$: A binary association relation for tone and TBU
  - $p$ and $s$: the immediate *predecessor* and *successor*
  - $p^*$ and $s^*$: starred *predecessor* and *successor* to select particular elements such as metrically strong TBUs and phrasal boundaries.

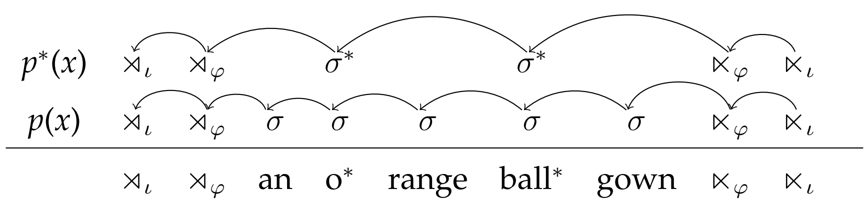



Table 2: A metrical grid using a tier-based representation.

- These string and relational models are defined with *first order* (FO) predicate logic.
  - $\sigma(x)$ for each $\sigma \in \Sigma$ denotes *atomic predicates* which are true when $x$ is interpreted as a positions in the unary relation $P_\sigma$ of a model.

Two transductions:

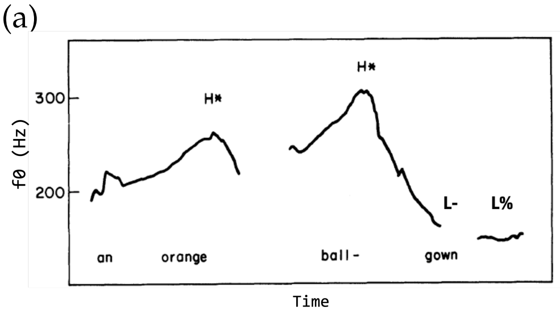
1. MELODIC TRANSDUCTION: making slots for the tones (unspecified T) and associate them with their TBUs.
2. MEANING TRANSDUCTION: filling in the actual Hs and Ls.

## 4.1 American English

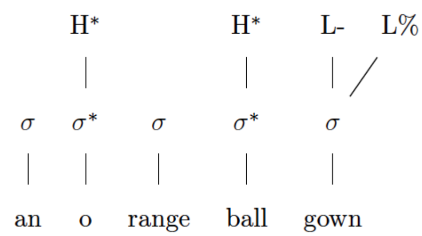
### 4.1.1 Basic intonational pattern

- American English is a *head-prominence* intonational (post-lexical pitch accent) language, where *metrically strong positions* receive pitch accents in a phrase.
- That is, the accented syllables (i.e., $\sigma^*$) are associated with pitch accents (e.g., H*, L*, H*+L, H+L*, L*+H, L+H*).
- A basic prosodic unit for these pitch accents are an intermediate intonational phrase (ip), where a phrase tone (e.g., L-, H-) are associated at the right edge of an ip.
- More than one ip can group together to become an intonational phrase (IP), in which a boundary tone (e.g., L%, H%) is associated with the right edge (or the left edge) of an IP.

(a)



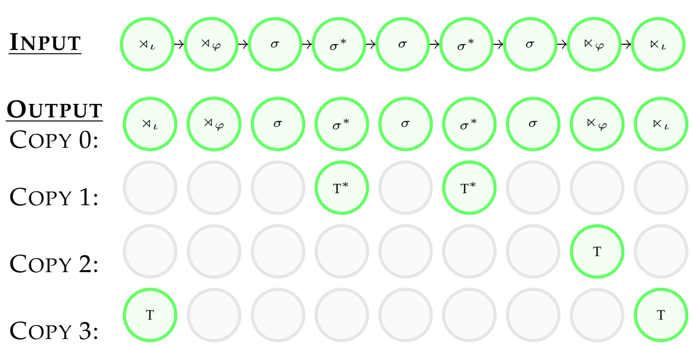

(b)




### 4.1.2 Melodic transduction

**Step 1: Copying** Tones in American English are literal copies of starred syllables and phrasal boundaries.

| | | |
|---|---|---|
| **Syllable copying** | $\sigma^0_{\text{o}}(x) = \sigma_{\text{i}}(x)$ | $\sigma^{*0}_{\text{o}}(x) = \sigma^*_{\text{i}}(x)$ |
| **Boundary copying** | $\ltimes_{\varphi\text{o}}^{0}(x) = \ltimes_{\varphi\text{i}}(x)$ | $\ltimes_{\iota\text{o}}^{0}(x) = \ltimes_{\iota\text{i}}(x)$ |
| | $\rtimes_{\varphi\text{o}}^{0}(x) = \rtimes_{\varphi\text{i}}(x)$ | $\rtimes_{\iota\text{o}}^{0}(x) = \rtimes_{\iota\text{i}}(x)$ |
| **Tonal copying** | $\text{T}^{*1}_{\text{o}}(x) = \sigma^*_{\text{i}}(x)$ | |
| | $\text{T}^2_{\text{o}}(x) = \ltimes_{\varphi\text{i}}(x)$ | |
| | $\text{T}^3_{\text{o}}(x) = \ltimes_{\iota\text{i}}(x) \vee \rtimes_{\iota\text{i}}(x)$ | |

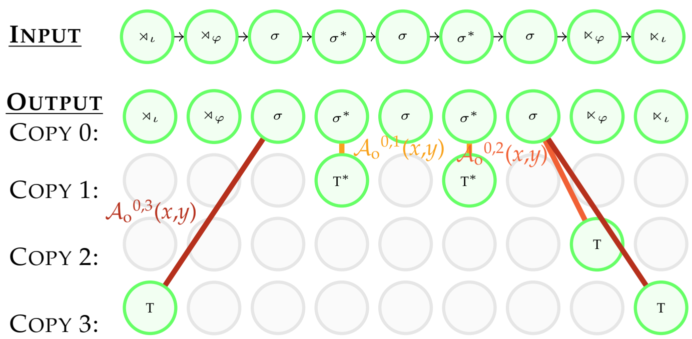


**Step 2: Tone-TBU association**

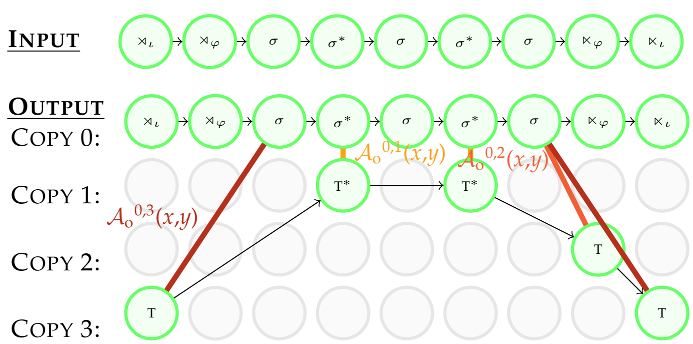
$$\mathcal{A}_{\mathrm{o}}{}^{0,1}(x,y) = \mathcal{A}_{\mathrm{o}}{}^{1,0}(y,x) \stackrel{\text{def}}{=} x \approx y$$

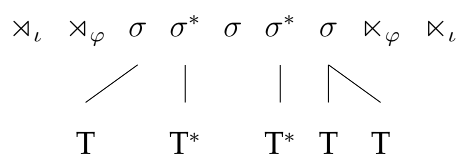
$$\mathcal{A}_{\mathrm{o}}{}^{0,2}(x,y) = \mathcal{A}_{\mathrm{o}}{}^{2,0}(y,x) \stackrel{\text{def}}{=} \sigma_{\mathrm{i}}(x) \ \wedge \ \ltimes_{\varphi\mathrm{i}}(y) \ \wedge \ y \approx s(x)$$

$$\mathcal{A}_{\mathrm{o}}{}^{0,3}(x,y) = \mathcal{A}_{\mathrm{o}}{}^{3,0}(y,x) \stackrel{\text{def}}{=} (\sigma_{\mathrm{i}}(x) \wedge \rtimes_{\iota\mathrm{i}}(y) \wedge y \approx p(p(x))) \vee (\sigma_{\mathrm{i}}(x) \wedge \ltimes_{\iota\mathrm{i}}(y) \wedge y \approx s(s(x)))$$



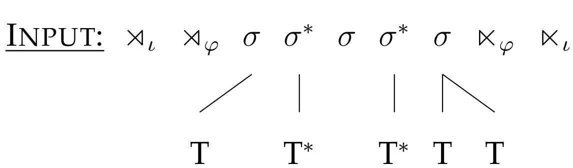

**Step 3: Defining the order in melodies** Note that the output precedence relations are defined automatically from the input precedence relations.

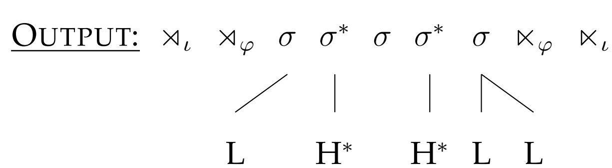



### 4.1.3 Declarative transduction – H* H* L L%

- $\mathcal{S}_{\mathrm{i}} = \{\sigma, \sigma^*, \ltimes_\varphi, \ltimes_\iota, \rtimes_\varphi, \rtimes_\iota, \mathrm{T}, \mathrm{T}^*\}$
- $\mathcal{S}_{\mathrm{o}} = \{\sigma, \sigma^*, \ltimes_\varphi, \ltimes_\iota, \rtimes_\varphi, \rtimes_\iota, \mathrm{H}, \mathrm{L}, \mathrm{H}^*, \mathrm{L}^*, \mathrm{L{+}H}^*, \mathrm{L}^*{+}\mathrm{H}, \mathrm{H}^*{+}\mathrm{L}, \mathrm{H{+}L}^*\}$
- We can change the input Ts with L tones and the input T*s with H tones, using these formulas for the declarative: $\mathrm{H}^*{}_{\mathrm{o}}(x) = \mathrm{T}^*{}_{\mathrm{i}}(x)$ and $\mathrm{L}_{\mathrm{o}}(x) = \mathrm{T}_{\mathrm{i}}(x)$.



### 4.1.4 Summary

- The melodies in the output were the direct reflections of the heads of the prosodic unit – starred syllables.
- Crucially, by copying the starred syllables, the head-prominence characteristic in American English intonation was captured.
- Also, the tone-TBU associations were defined *locally* from the input structure without using any quantifiers.
- Therefore, we were able to see that the head-prominence intonational pattern in American English is a QF logical interpretation of a metrical and prosodic structure.

## 4.2 Seoul Korean

### 4.2.1 Basic intonational pattern

- Seoul Korean is an *edge-prominence* intonational language (Jun, 2006), where phrasal boundaries are marked with prominence without any pitch accents.
- Importantly, a basic phrasal unit in Seoul Korean is an Accentual Phrase (AP), which has a typical tonal pattern LH...LH and typically consists of three or four syllables.
- But when the initial segment of an AP is an aspirated or a tense consonant, the tonal pattern becomes HH...LH. Thus, the tonal pattern of Korean AP is either LH...LH or HH...LH.

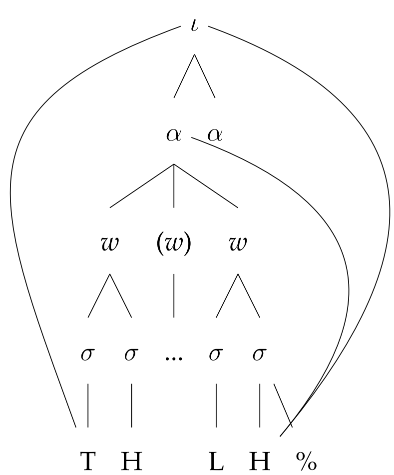
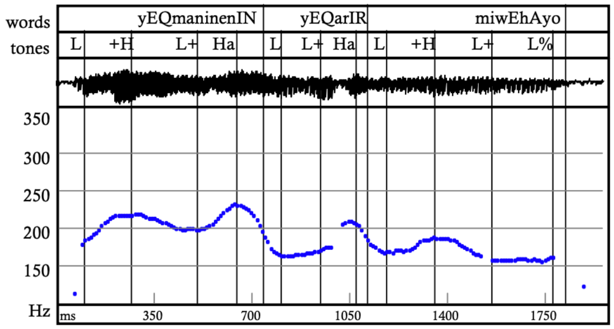
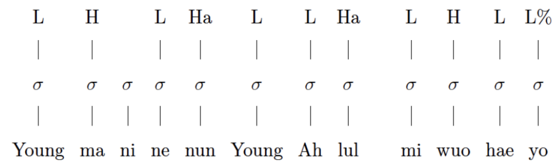



Figure 2: Intonational structure in Seoul Korean (Jun, 2006).

(a) (b)

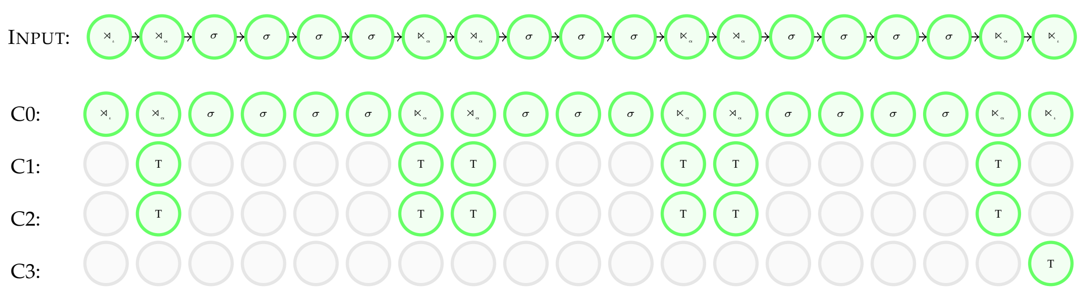



### 4.2.2 Melodic transduction

**Step 1: Copying** Tones in Seoul Korean are literal copies of elements in the prosodic structure, which are only phrasal boundaries.

| | | |
|---|---|---|
| **Syllable copying** | $\sigma_{\text{o}}^{0}(x) = \sigma_{\text{i}}(x)$ | $\sigma_{\text{F}\text{o}}^{0}(x) = \sigma_{\text{Fi}}(x)$ (F = [+stiff])[1] |
| **Boundary copying** | $\ltimes_{\alpha\text{o}}^{0}(x) = \ltimes_{\alpha\text{i}}(x)$ | $\ltimes_{\iota\text{o}}^{0}(x) = \ltimes_{\iota\text{i}}(x)$ |
| | $\rtimes_{\alpha\text{o}}^{0}(x) = \rtimes_{\alpha\text{i}}(x)$ | $\rtimes_{\iota\text{o}}^{0}(x) = \rtimes_{\iota\text{i}}(x)$ |
| **Tonal copying** | $\text{T}_{\text{o}}^{1}(x) = \rtimes_{\alpha\text{i}}(x) \vee \ltimes_{\alpha\text{i}}(x)$ | |
| | $\text{T}_{\text{o}}^{2}(x) = \rtimes_{\alpha\text{i}}(x) \vee \ltimes_{\alpha\text{i}}(x)$ | |
| | $\text{T}_{\text{o}}^{3}(x) = \ltimes_{\iota\text{i}}(x)$ | |



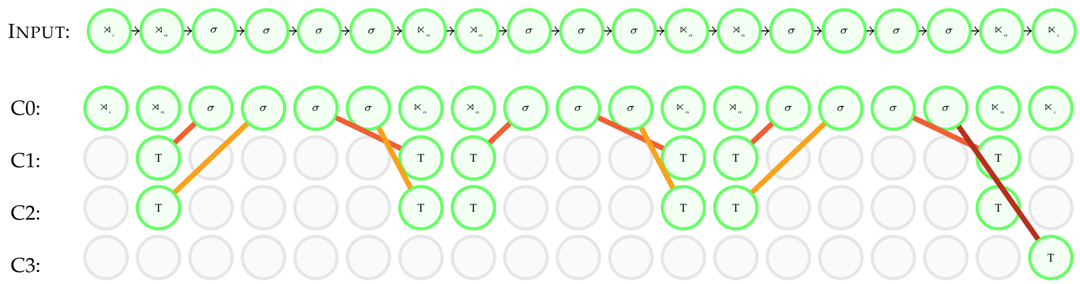

[1]Note that for Seoul Korean, the stiffness feature for aspirated or tense consonants is specified in the syllable such that this featural information can be retrieved later when computing the actual tonal contour, HH...LH, as shown in $\sigma_{\text{F}\text{o}}^{0}(x) = \sigma_{\text{Fi}}(x)$ (F = [+stiff]).

**Step 2: Tone-TBU association**

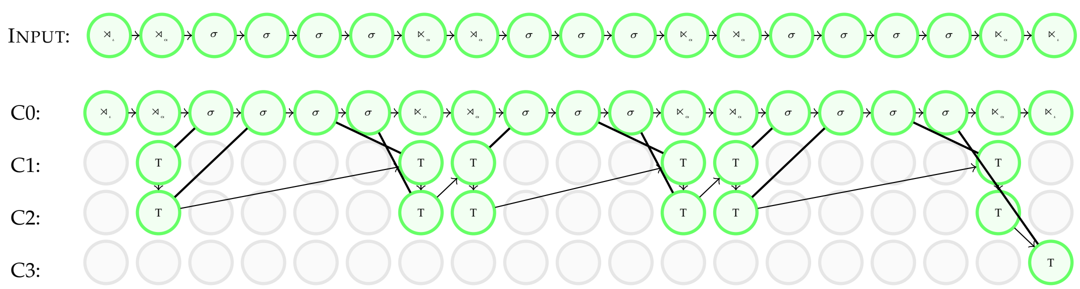
$$\mathcal{A}_o{}^{0,1}(x,y) = \mathcal{A}_o{}^{1,0}(y,x) \stackrel{\text{def}}{=} \sigma(x) \wedge (\rtimes_\alpha(y) \wedge\ y \approx p(x)) \vee (\ltimes_\alpha(y) \wedge\ y \approx s(s(x)))$$

$$\mathcal{A}_o{}^{0,2}(x,y) = \mathcal{A}_o{}^{2,0}(y,x) \stackrel{\text{def}}{=} \sigma(x) \wedge \neg(\ltimes_\iota(y) \wedge y \approx s(s(x))) \wedge (\rtimes_\alpha(y) \wedge\ y \approx p(p(x))) \vee (\rtimes_\alpha(y) \wedge y \approx s(s(x)))$$

$$\mathcal{A}_o{}^{0,3}(x,y) = \mathcal{A}_o{}^{3,0}(y,x) \stackrel{\text{def}}{=} \sigma(x) \wedge \ltimes_\iota(y) \wedge y \approx s(s(x))$$

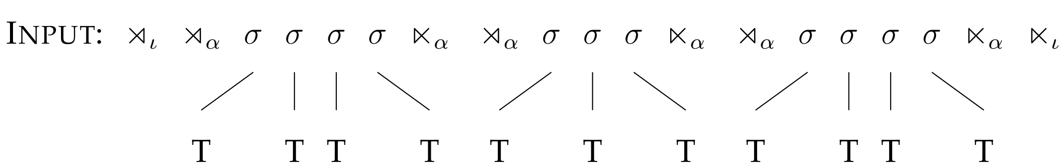


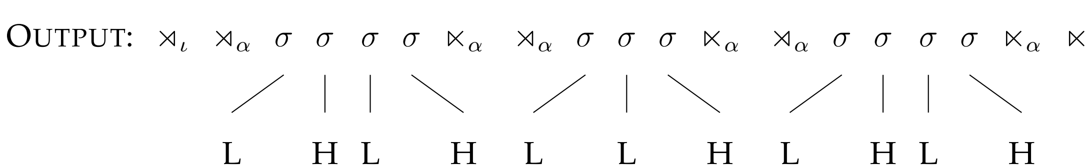

**Step 3: Defining the order in melodies**




#### 4.2.3 Declarative transduction – LHLHa LLHa HLHL%

- $\mathrm{L_o}(x) = \mathrm{T_i}(x)\ \wedge\ (\rtimes_\alpha(p(x))\ \vee \ltimes_\alpha(s(s(x))))$
- $\mathrm{H_o}(x) = \mathrm{T_i}(x)\ \wedge\ (\rtimes_\alpha(p(p(x)))\ \wedge\ \neg\mathrm{H}(s(x)))\ \vee \ltimes_\alpha(s(x))$



### 4.2.4 Summary

- The melodies in the output were literal copies of *only* boundaries in the input structure.
- Unlike the head-prominence intonational pattern in American English, copying only boundaries was able to capture the edge-prominence characteristic in the intonation of Seoul Korean, showing that the edge tones were the direct reflections of the phrasal edges.
- Just like the local association in American English, the tone-TBU associations were defined *locally* from the input structure without using any quantifiers.
- Therefore, we were able to see that the edge-prominence intonational pattern in Seoul Korean is a QF logical interpretation of a metrical and prosodic structure.

## 4.3 Tokyo Japanese

### 4.3.1 Basic intonational pattern

- Tokyo Japanese is a *lexical pitch accent language* (Beckman & Pierrehumbert, 1986), where tones are lexically specified for particular moras, while other tones are defined in the phrase-level.
- A basic phrasal unit in Tokyo Japanese is an Accentual Phrase (AP), which is characterized with an initial rising pitch accent at the beginning of an AP.

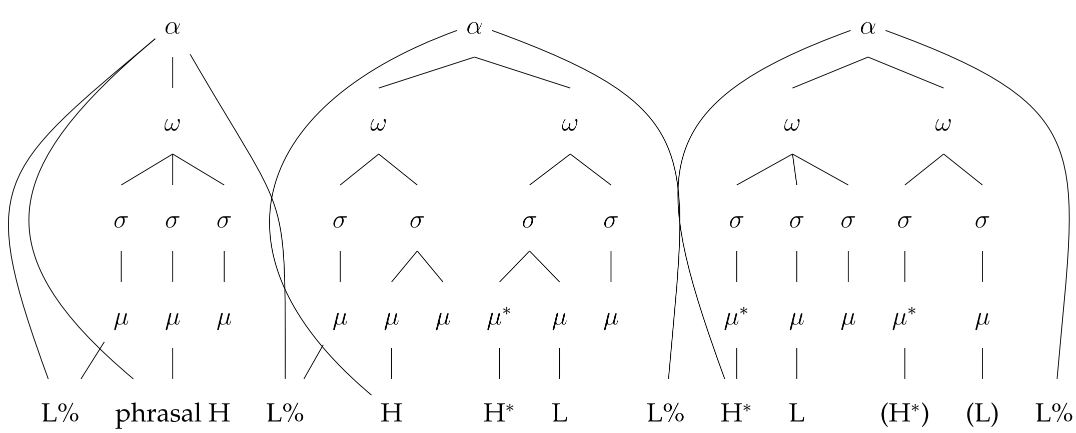


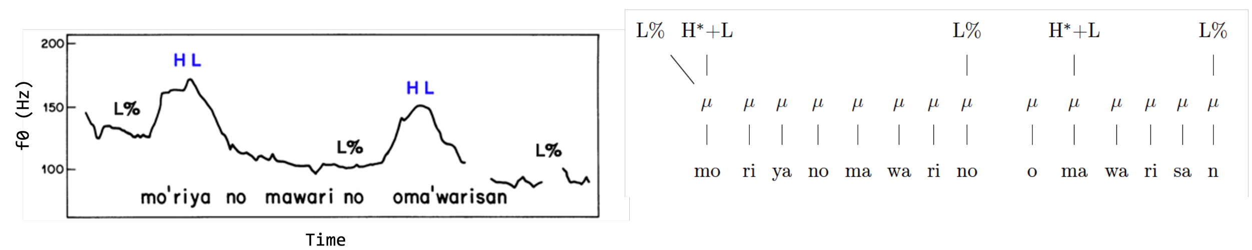

Figure 3: Intonational structure in Tokyo Japanese.

1. When the first syllable of the first lexical item in an AP is *accented* (e.g., <u>ká</u>geboosi), H*L is associated to the first mora of the accented syllable. Due to the realization of the lexical pitch accent on the first and second moras, linking a phrasal H tone to the

second mora is blocked. L% boundary tone in the preceding AP is associated to the final mora of the preceding AP, instead of being associated to the first mora of the AP.

2. When the first syllable of the first lexical item in an AP *unaccented* (e.g., toomórokoski, moosikomi), a phrasal H tone is usually linked to the second sonorant mora and L% boundary tone of the preceding AP is associated to the first mora of the following AP.
3. Lastly, L% boundary tone is inserted at the beginning of the utterance as a whole. A postlexical rule deletes all accents after the first accent in an AP, which is known as deaccentuation.

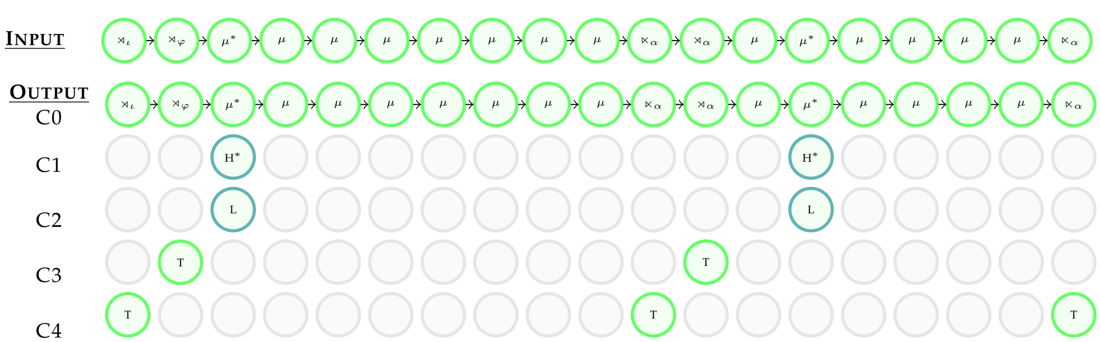
(a) (b)




### 4.3.2 Melodic transduction

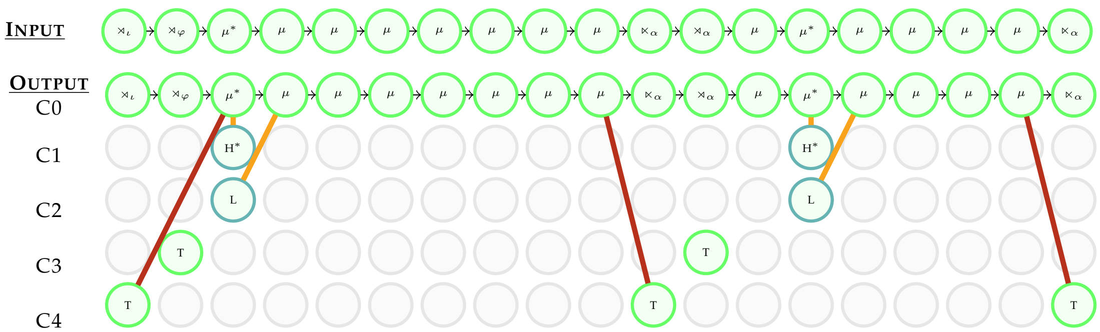
**Step 1: Copying** Note that the starred moras are directly reflected to the lexical pitch accents (H*L); while the phrasal boundaries are reflected to unspecified post-lexical tones (T).

| | | |
|---|---|---|
| **Mora copying** | $\mu_{\text{o}}^{0}(x) = \mu_{\text{i}}(x)$ | $\mu^{*}{}_{\text{o}}^{0}(x) = \mu^{*}{}_{\text{i}}(x)$ |
| **Boundary copying** | $\ltimes_{\alpha\text{o}}^{0}(x) = \ltimes_{\alpha\text{i}}(x)$ | $\ltimes_{\iota\text{o}}^{0}(x) = \ltimes_{\iota\text{i}}(x)$ |
| | $\rtimes_{\alpha\text{o}}^{0}(x) = \rtimes_{\alpha\text{i}}(x)$ | $\rtimes_{\iota\text{o}}^{0}(x) = \rtimes_{\iota\text{i}}(x)$ |
| **Tonal copying** | $\text{H}^{*}{}_{\text{o}}^{1}(x) = \mu^{*}{}_{\text{i}}(x)$ | $\text{L}_{\text{o}}^{2}(x) = \mu^{*}{}_{\text{i}}(x)$ |
| | $\text{T}_{\text{o}}^{3}(x) = \rtimes_{\alpha\text{i}}(x)$ | $\text{T}_{\text{o}}^{4}(x) = \rtimes_{\iota\text{i}}(x) \wedge \ltimes_{\alpha\text{i}}(x)$ |

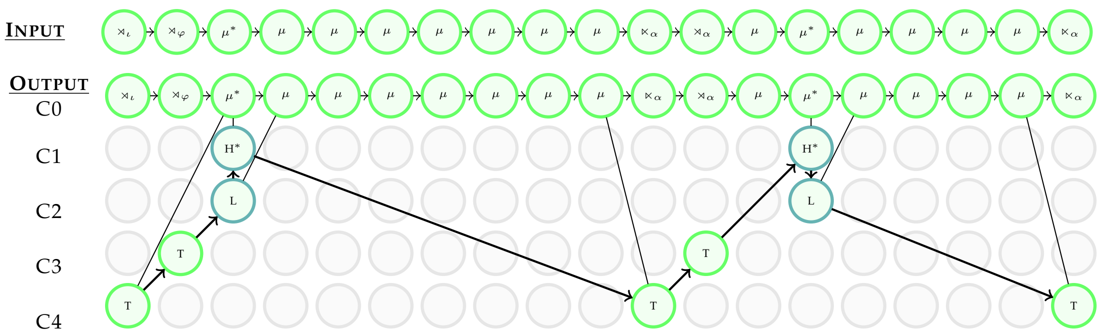


### Step 2: Tone-TBU association

$$\mathcal{A}_o{}^{0,1}(x,y) = \mathcal{A}_o{}^{1,0}(y,x) \stackrel{\text{def}}{=} \mu_i{}^*(x) \wedge \rtimes_{\alpha i}(y) \wedge y \approx p^*(x)$$
$$\mathcal{A}_o{}^{0,2}(x,y) = \mathcal{A}_o{}^{2,0}(y,x) \stackrel{\text{def}}{=} \mu_i(x) \wedge \rtimes_{\alpha i}(y) \wedge y \approx p^*(x)$$
$$\mathcal{A}_o{}^{0,3}(x,y) = \mathcal{A}_o{}^{3,0}(y,x) \stackrel{\text{def}}{=} \mu_i(x) \wedge (\rtimes_{\alpha i}(y) \wedge y \approx s(s(x))) \wedge \neg(\mu^*(y) \wedge y \approx s(x))$$
$$\mathcal{A}_o{}^{0,4}(x,y) = \mathcal{A}_o{}^{4,0}(y,x) \stackrel{\text{def}}{=} \mu_i(x) \wedge (\rtimes_{\iota i}(y) \vee \rtimes_{\alpha i}(y) \wedge y \approx p(p(x))) \vee (\ltimes_{\alpha i}(y) \wedge y \approx s(x))$$

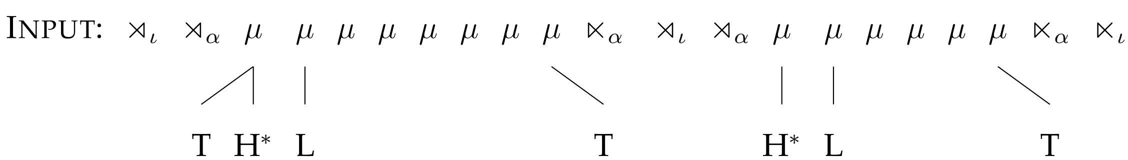


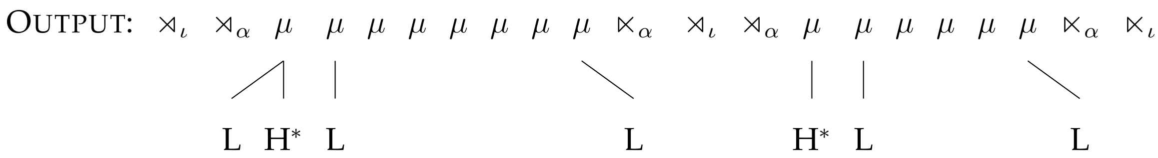

### Step 3: Defining the order in melodies




### 4.3.3 Declarative transduction

- $\mathcal{S}_i = \{\mu, \mu^*, \ltimes_\alpha, \ltimes_\iota, \rtimes_\alpha, \rtimes_\iota, \mathrm{T}, \mathrm{H}^*, \mathrm{L}\}$
- $\mathcal{S}_o = \{\mu, \mu^*, \ltimes_\alpha, \ltimes_\iota, \rtimes_\alpha, \rtimes_\iota, \mathrm{H}^*, \mathrm{H}, \mathrm{L}\}$
- $\mathrm{L}_o(x) = \mathrm{T}_i(x)$






#### 4.3.4 Summary

- The melodies in the output were literal copies of starred moras and boundaries in the input structure.
- Unlike the post-lexical (head-prominence and edge-prominence) intonational patterns in American English and Seoul Korean, copying starred moras directly to H* and L was able to capture the lexically specified pitch accent in Tokyo Japanese.
- Also, copying boundaries was able to capture the realization of post-lexical tones in addition to the lexical pitch accents.
- These lexical and post-lexical pitch accent patterns in Tokyo Japanese captures the typical initial rising pitch accent in an AP in Tokyo Japanese.
- Moreover, even considering the deaccentuation pattern that only selects the first lexical pitch accent in an AP, the tone-TBU associations were defined *locally* from the input structure without using any quantifiers.
- Thus, we can also conclude that the lexical pitch accent pattern in Tokyo Japanese is a QF logical interpretation of a metrical and prosodic structure.

## 5 Discussion

1. How can we define the autosegmental representation of intonation as a logical interpretation and how does this say about the computational nature of intonation?
    - Intonation is a *QF logical interpretation of a metrical and prosodic structure*, which can be defined *locally* from the input structure.
    - Copying what kind of prosodic elements from the input to the output leads to the characterization of different metrical and prosodic realizations in intonation.
    - Tone-TBU association in intonation turned out to be a local process without any quantifiers.
2. How can we posit a computational theory of intonation that predicts a typology of possible intonational patterns of languages based on the computational nature?

- By defining the intonational structure as a QF logical interpretation of a metrical and prosodic structure that are input strictly local, we were able to create an *intonational theory* that is restrictive enough and more explicit to characterize different intonational patterns of the languages.
- However, further research is needed to generalize these results by examining other languages that fall into the same intonational categories: e.g., Spanish, French, Lekeitio Basque, etc.

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