BMRS and Underspecification

Motivation

> "At the heart of debates on underspecification are assumptions about the nature of the representations and the nature of the rule system." - Mohanan (1991, p. 323)

> "Since the target of a spreading rule must be unspecified for the spreading feature in order to be a target in the first place, this feature is "predictable" by the spreading rule only insofar as it is unspecified in the target. However, recall that a feature is unspecified in underlying representations only insofar as it is predictable by a rule! Under this approach, spreading rules and input underspecification are interdependent in such a way that the function of each at best largely duplicates the function of the other." - Baković (2000, p. 301)

> "...a full binary feature system is the only possible result when using logical negation. Consequently, in order to effectively have 0 values, the positive/negative feature valuation must be encoded into the representational primitives rather than emerge from the logical connectives." Nelson (2022, p. 2)

The goal of this work is to understand underspecification in phonology as a computational property rather than a representational property.[1] To begin, let's consider how we can frame what underspecification is and why underspecification is used.

What: Underspecification is the idea that a linguistic representation can have missing information: some elements of the representation that could be specified, but aren't.

Why: Underspecification is used to explain why certain phonological elements are not a target or trigger for a phonological process.

In the former case, underspecification is clearly a representational property. In the latter case, representation is used as an *intensional* explanation for for certain phonological maps. But these maps exist *extensionally* and therefore can be described without underspecified representations. The goal of this work is to look for a shared computational structure that can explain these types of maps that does not rely on a specific feature encoding.

[1] The primary focus is on *featural* underspecification. This is joint work with Eric Baković.

The Origin of Skepticism

Previously (Nelson, 2022) I looked at natural class formation based on various types of feature primitives and different logics. The logics were restricted to subsets of QF with only conjunction: Conjunction of Positive Literals (CPL) and Conjunction of Negative Literals (CNPL). This was due to the following ways that natural class formation is described:

> "...a group of sounds in an inventory which share one or more distinctive features, *within a particular feature theory* to the exclusion of all other sounds in the inventory..." (Mielke, 2008, p. 12)
>
> "...an adequate feature system should permit any natural class of sounds to be represented by the conjunction of features in a matrix..." (Kenstowicz and Kisseberth, 1979, p. 241)
>
> "Natural classes can be defined in terms of conjunctions of features..." (Odden, 2005, p. 49)

The literals (primitives) were varied between univalent literals (essentially privative features) and bivalent literals (encoding +/- into the representation. I then tested what types of natural classes were generated by altering the logic and representations.

1. CPL w/ univalent literals correctly represent privative underspecification feature systems but undergenerate for full binary feature systems and contrastive underspecification feature systems.
2. CNPL w/univalent literals correctly represent full binary features systems but overgenerate for both types of underspecification systems.
3. CPL w/ bivalent primitives correctly represent all three types of feature systems.

An underspecified domain element in the last case would satisfy $\neg[+f \vee -f]$. This is still somewhat unsatisfying in that you can you use this as a third value to describe aspects of the structural description of a process. Stanley (1967, 410, emphasis added) warns against this:

> The correctness of any empirical claim that distinctive features are binary is, of course, not at issue here. *The point is simply that, once we decide to use a binary system, we must be formally consistent.* Unfortunately, it is all too easy to be formally inconsistent by letting '0' [i.e. a blank cell – EB/SN] function as a third feature value, and this has often been done unknowingly in the writing of generative grammars. *What is important is that we keep the meaning of '0' clearly in mind. It is not a feature value, but merely a mark which indicates that the feature value of the entry in which it appears has not yet been filled in.*

Underspecification without Underspecified Representations

In Nelson and Baković (2024) we used Russian voicing assimilation to show that we can change the framing of underspecification from "segments which *cannot* be targeted" by a process to "segments" which *are not* targeted" by a process. Ultimately, what we point out is that underspecification serves to remove certain types of elements as targets and triggers, and because of redundancy rules, the valuation of the underspecified feature does not need to play a role at all.

In Russian, obstruents contrast in voicing and also participate as both triggers and targets for a voicing assimilation process. Sonorants, on the other hand, neither contrast nor participate as either a trigger or target in the process.

One way to write the rule is:

$$[-\text{son}] \rightarrow [\alpha\text{voi}] \ / \ \_ \begin{bmatrix} \alpha\text{voi} \\ -\text{son} \end{bmatrix}$$

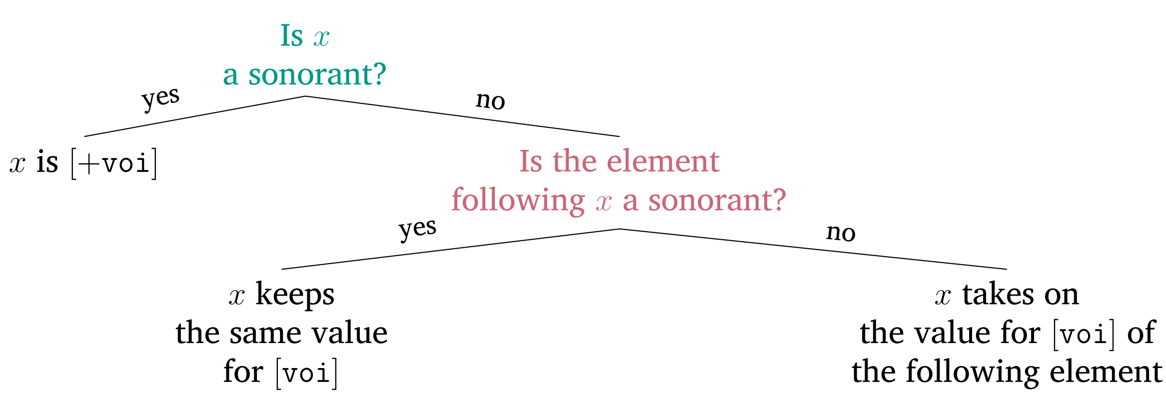
Capitalizing on sonorants not being contrastive for voicing in the language, a simpler (?) analysis can be head if we say that sonorants are not specified for the [voice] feature and have the following rule:

$$x \rightarrow [\alpha\text{voi}] \ / \ \_ \ [\alpha\text{voi}]$$

This of course would also then require the redundancy rule:

$$[+\text{son}] \rightarrow [+\text{voi}]$$

Given these generalizations, we point out that the decision tree below provides a way to determine the voicing property of an output element. The first query removes sonorants from the set of targets and the second query removes them from the set of triggers.



Decision trees are really just IF...THEN...ELSE syntax, which means we can use Boolean Monadic Recursive Schemes (BMRS; Bhaskar et al., 2020; Chandlee and Jardine, 2021; Bhaskar et al., 2023) to formalize this idea. The decision tree above is equivalent to the BMRS function below.

(1) $\phi_{\texttt{voi}}(x) :=$ (Russian voicing function)

$\texttt{IF son}(x)\ \texttt{THEN}\ \top$

$\texttt{ELSE IF son}(\texttt{s}(x))\ \texttt{THEN voi}(x)$

$\texttt{ELSE voi}(\texttt{s}(x))$

Based on this insight, we proposed four properties for what we called UNDERSPECIFICATION MAPS.

(2) a. The map will define input-output conditions for the "underspecified feature".

b. Any underspecification map will include a nested conditional BMRS term.

c. Both the upper conditional P and lower conditional Q will determine a truth value based on the antecedent of the redundancy rule that fills in the "underspecified feature".

d. P partitions the set of targets while Q partitions the set of triggers.

Russian voicing assimilation can be contrasted with Catalan voicing assimilation where sonorants are not targets but sonorant consonants are triggers. Since they are not targets, the first "redundancy rule" condition still removes them from the set of targets, but we must instead use a feature like syll to remove only non-consonantal sonorants from the set of triggers.

(3) $\phi_{\texttt{voi}}(x) :=$ (Catalan voicing function)

$\texttt{IF son}(x)\ \texttt{THEN}\ \top$

$\texttt{ELSE IF syll}(\texttt{s}(x))\ \texttt{THEN voi}(x)$

$\texttt{ELSE voi}(\texttt{s}(x))$

Changing the upper conditional to $\texttt{syll}(x)$ would result in a computational structure satisfying (2) but would erroneously include sonorant consonants in the set of targets and therefore incorrectly describe the Catalan assimilation map. Likewise, changing the lower conditional to $\texttt{son}(\texttt{s}(x))$ would remove sonorant consonants from the set of triggers and once again describe an incorrect map for Catalan.[2]

This accounts for contrastive underspecification, but underspecification has been used other ways. Inkelas et al. (1997) consider another, distinct use of underspecification in their analysis of voicing in Turkish, in which three relevant classes of morphemes that end:

(a) in non-alternating voiceless stops,

(b) in non-alternating voiced stops, and

(c) in stops that alternate between voiceless (in codas) and voiced (elsewhere).

The authors propose that the final stops of morphemes in each of these classes are represented as in (4), with strictly feature-filling processes handling the eventual valuations of the alternating stops in class (c).

(4) a. $[-\texttt{voice}]$: $[\text{devle}\underline{\text{t}}] \sim [\text{devle}\underline{\text{t}}\text{i}]$ 'state ~ ACC'

b. $[+\texttt{voice}]$: $[\text{ety}\underline{\text{d}}] \sim [\text{ety}\underline{\text{d}}\text{y}]$ 'study ~ ACC'

c. $[0\,\texttt{voice}]$: $[\text{kana}\underline{\text{t}}] \sim [\text{kana}\underline{\text{d}}\text{ɯ}]$ 'wing ~ ACC'

The strictly feature-filling nature of the processes required by this analysis is a challenge for our approach. However, note that underspecification in this case is essentially being used as a lexical class diacritic.Indeed, the clearly exceptional $[+\texttt{voice}]$ class (b) consists mostly of loans like $[\text{etyd}]$ 'study' and $[\text{katalog}]$ 'catalog'. Suppose instead that the final stops of class (b) are underlyingly specified with some feature $[+f]$. The relevant facts can then be captured with the following BMRS function.

(5) $\phi_{\texttt{voi}}(x) :=$ (Turkish voicing function)

$\texttt{IF } f(x) \lor \texttt{son}(x) \texttt{ THEN } \top$

$\texttt{ELSE IF coda}(x) \land \texttt{stop}(x) \texttt{ THEN } \bot$

$\texttt{ELSE voi}(x)$

[2] One aspect left for clarification is the role of {sub,sup}er set relations. The function in (3) doesn't require $[+\,\texttt{syllabic}]$ elements to have a voicing specification since they are removed from the set of targets by being a subset of $[+\,\texttt{sonorant}]$ in the upper conditional and then removed from the set of triggers in the lower conditional.

The upper conditional ensures that morpheme-final stops in class (b), and sonorants generally, always surface as [+voice], making the underlying voicing value for class (b) irrelevant. The lower conditional is the standard coda devoicing function.

This is essentially the 'co-phonology' approach considered (and rejected) by Inkelas et al., whereby classes (a) and (c) result from a standard coda devoicing grammar while class (b) results from a grammar without devoicing.

- One reason they reject this approach is the existence of morphemes with voiced internal coda stops but alternating final stops, e.g. [edʒdaːt] ∼ [edʒdaːdɯ] 'ancestry ∼ ACC'. The problem is that co-phonologies apply to entire morphemes, while underspecification can be selectively applied to individual elements of morphemes.
- A second reason is due to a worry about an over-generating proliferation of co-phonologies.
- Our single BMRS function avoids both these issues.

The final interesting case has to do with discussions of coronal underspecification.

Two phonological maps of interest are:

(i) nasal place assimilation, a postlexical process targeting only coronal nasals (*i*[m] *Port Jeff*; *i*[ŋ] *Canada*),

(ii) /s/-voicing, a lexical process that targets only coronal /s/ and turns it to [z] after a tense/long vowel and before a vowel or a glide (*advi*[s]*e* ∼ *advi*[z]*ory*; *re*[f] ∼ *re*[f]*er*).

These two processes are noteworthy for the following reason: /s/-voicing requires direct reference to [Coronal] (Mohanan, 1991) while nasal place assimilation appears to require representational underspecification of [Coronal] (Avery and Rice, 1989). But since nasal place assimilation is postlexical, it requires [Coronal] to be underspecified at that late point in the derivation. How then could the early lexical process of /s/-voicing target a class of only [Coronal] sounds if it is underspecified until late in the derivation? We argue that the feature [Coronal] is always representationally fully specified and that its "underspecified" behavior late in the derivation emerges from properties of the computation. In (6) we show a BMRS function describing how the output property of place is computed in terms of input properties for the nasal place assimilation map.

(6) $\phi_{\texttt{place}}(x) :=$ (English place function)

$\texttt{IF place}(x) \in \{\texttt{lab}, \texttt{dor}\}\ \texttt{THEN place}(x)$

$\texttt{ELSE IF nas}(x)\ \texttt{THEN IF stop}(\texttt{s}(x)) \wedge \texttt{place}(\texttt{s}(x)) \in \{\texttt{lab}, \texttt{dor}\}$

$\texttt{THEN place}(\texttt{s}(x))\ \texttt{ELSE cor ELSE cor}$

We assume $\texttt{place} : D \rightarrow \{\texttt{lab}, \texttt{cor}, \texttt{dor}\}$ is a function from domain elements to major place features. This more closely aligns with the feature geometric analysis given by Avery and Rice (1989) which views the place node as having one of these properties. We also abuse notation by using $\in$ as a shorthand for disjunction (which is itself a shorthand within the BMRS syntax). Note that none of the statements in the function check if an input element has the property `cor`. Consequently, the output property for $[\texttt{Coronal}]$ for a given segment is determined regardless of its representational specification for $[\texttt{Coronal}]$ in the input. It ultimately is an "elsewhere" case.

Note that (6) satisfies the conditions for an UNDERSPECIFICATION MAP: (a) it defines input-output conditions for `place`, the "underspecified feature"; (b) it includes a nested conditional; (c) both the upper conditional P (red) and lower conditional Q (blue) determine a truth value based on the antecedent features of a redundancy rule $[-\texttt{lab}, -\texttt{dor}] \rightarrow [+\texttt{cor}]$; and (d) P partitions the set of targets and Q partitions the set of triggers.

While the computational structure thus makes the input specification of $[\texttt{Coronal}]$ arbitrary, recall that the contradictory case of /s/-voicing requires full specification early in the derivation. So, as it turns out, this is not a contradiction at all: the arbitrariness of $[\texttt{Coronal}]$ specification in the map in (6) equally supports full specification on the input. **?** point out several other examples for which $[\texttt{Coronal}]$ must be fully specified in English, including morpheme structure constraints, loanword adaptation patterns, and phonotactic constraints. If underspecification is an epiphenomenal property of certain computations, then these issues disappear.

The Beginning of a Typology

Essentially, we can think about a typology where certain types of segments are included as targets or triggers. When the class is both targets and triggers this is basically full specification. When they are only targets but not triggers, it corresponds with autosegmental spreading uses of underspecification. When they are only triggers but not targets, it corresponds with the diacritic/prespecification uses of underspecification. When they are not targets nor triggers, it corresponds with full constrastive underspecification.

There's also a parallel with our account of underspecification suggesting the underlying specification doesn't matter and richness of the base. In the following examples, we consider the spreading of [voice] again.

Regressive voicing assimilation between obstruents (/td/ $\mapsto$ [dd]; /dt/ $\mapsto$ [tt]). How do sonorants behave, assuming base richness (/d, t, n, n̥/) but voicing redundancy?

target & trigger	*target only*	*trigger only*	*neither*	
/tn/ $\mapsto$ [dn]	/tn/ $\mapsto$ [tn]	/tn/ $\mapsto$ [dn]	/tn/ $\mapsto$ [tn]	
/nt/ $\mapsto$ [n̥t]	/nt/ $\mapsto$ [n̥t]	/nt/ $\mapsto$ [nt]	/nt/ $\mapsto$ [nt]	
$\varphi_{\texttt{voi}}(x) :=$	$\varphi_{\texttt{voi}}(x) :=$	$\varphi_{\texttt{voi}}(x) :=$	$\varphi_{\texttt{voi}}(x) :=$	
IF final(x)	IF final(x)	IF final(x)	IF final(x)	specify potential trigger
THEN	THEN	THEN	THEN	
IF son(x)	IF son(x)	IF son(x)	IF son(x)	
THEN $\top$	THEN $\top$	THEN $\top$	THEN $\top$	redundancy rule
ELSE voi(x)	ELSE voi(x)	ELSE voi(x)	ELSE voi(x)	faithful
	ELSE	ELSE	ELSE	
		IF son(x)	IF son(x)	
		THEN $\top$	THEN $\top$	redundancy rule
			ELSE	
	IF son(s(x))		IF son(s(x))	
	THEN voi(x)		THEN voi(x)	faithful
ELSE $\varphi_{\texttt{voi}}$(s(x))	ELSE $\varphi_{\texttt{voi}}$(s(x))	ELSE $\varphi_{\texttt{voi}}$(s(x))	ELSE $\varphi_{\texttt{voi}}$(s(x))	spreading

- The first IF handles domain elements that cannot be targets of the process and therefore must have their voicing values stipulated. Sonorants are always voiced while obstruents maintain their underlying voicing contrast. Since the triggering environment is on the right (regressive assimilation) it is the final element in the domain. If the triggering element were on the left (progressive assimilation) it would be the first element.
- Column one (*target & trigger*) has a single ELSE statement that is the implementation of spreading. There are no stipulations because everything goes here.
- Second column (target only) has another embedded conditional ensuring sonorants are removed from the set of triggers and domain elements preceding them surface faithfully.
- Third column (trigger only) has another embedded conditional ensuring sonorants are removed from the set of targets thus making them both block and initiate spreading.
- Fourth column includes all of the aspects of the previous three columns.

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