# THE DERANGEMENT OF SUBREGULAR GRAMMARS*

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*Section 1 Introduction and Preliminaries*

Informally, a *derangement* is the complete scrambling of a set: it is a mapping imposed on all elements of a set such that none of the elements map to themselves. In this paper, we use the derangement and partial derangement of alphabets—and the grammars and languages constructed from these alphabets—to explore some of the predictions of subregular phonology (Heinz 2010, 2018; Graf 2022; among others). We focus on strictly local grammars (McNaughton & Papert 1971), using stress and accent patterns as the primary examples. We will discover that strict locality predicts a range of stress patterns that is immensely greater than the range that is actually attested.

The examination will focus on the alphabets, grammars, and forms of two "seed" patterns. The first is the stress pattern of Maranungku (Tryon 1970), a simple left to right, quantity-insensitive, trochaic stress pattern. It has neither adjacent stressed syllables, *clash*, nor adjacent stressless syllables, *lapse*.[1] The second is the stress pattern of Wergaia (Hercus 1986). The Wergaia pattern is similar to the Maranungku pattern, but it exhibits a limited type of quantity sensitivity. The two patterns will be used as "seed" patterns in this sense: we will shuffle the alphabetic symbols from which their grammars and forms are constructed to create new grammars and new forms constituting new patterns. From a linguistic perspective, each of the new patterns is distinct: the alphabetic symbols are distributed differently in each case. From a computational perspective, however, the new patterns are not distinct: they all have the same formal characteristics. While the assignment of symbols to distributions will vary, the distributions themselves will remain the same. The seed patterns are both strictly local, and the new patterns will also be strictly local. In fact, each new pattern will exhibit the same degree of locality as its original seed pattern.

We will examine each seed pattern, and its associated scrambled patterns, in the context of several different alphabets. Each alphabet will contain one to six more symbols than the previous alphabet, and it will be capable of capturing more fine-grained distinctions. The first alphabet will have just two symbols, merely distinguishing stressed syllables from unstressed syllables. The final alphabet will have twelve symbols, allowing it to distinguish between primary stressed syllables, secondary stressed syllables, and unstressed syllables, as well as distinguishing between light syllables and three types of heavy syllable.[2]

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[1] This stress pattern is fairly common. In addition to Maranungku, it is also found in Icelandic (Gussmann 1985; Árnason 1980, 1985), Ningil (Manning and Saggers 1977), Ono (Phinnemore 1985) and several others.

[2] While we will end our examination with the twelve-symbol alphabet, it should be noted that an alphabet with twelve symbols is insufficient to capture all the nuances in the stress patterns found among the world's languages.

The number of symbols that an alphabet contains is a key figure because it puts a cap on the number of distinct patterns that can be created by scrambling the original seed pattern's alphabet. The number of ways that an alphabet can be scrambled increases factorially with the alphabet's size. If an alphabet has *n* symbols, it can be scrambled in *n*! ways, so scrambling can result in at most *n*! distinct patterns. The actual number of distinct patterns that can be created from the scrambled alphabet of a particular seed pattern is sometimes smaller, however. We will see cases where the number of distinct patterns matches the number of possible scramblings, and we will see cases where the number of distinct patterns is smaller.

Since the largest alphabet that we will consider has twelve symbols, the maximum number of distinct patterns that might be created from a single seed pattern is 12!, or 479,001,600. With the Wergaia pattern as the seed pattern, the actual number of distinct patterns turns out to be 39,916,800. While each of these patterns is unique from a phonological perspective, they are not unique from a computational perspective. They are computationally identical. At the point in the discussion where we are considering 39,916,800 distinct stress patterns, the reader will likely be wondering how many of the 39,916,800 patterns are actually attested.[3] We will only give a rough answer to this question, but we can give it in advance. Heinz's (2007, 2009) database of stress and accent patterns is likely the most extensive and thorough at present. It contains a total of 109 distinct stress patterns that have been found among the world's languages, and we will assume that this is a roughly accurate count of the attested stress patterns. Given this assumption, no more than 109 of the 39,916,800 distinct stress patterns yielded through scrambling of the Wergaia seed pattern are actually attested.

At the heart of subregular phonology is the idea that phonology is implemented with a set of relatively weak formal mechanisms that restrict the complexity of phonological patterns. Subregular phonologists argue that most or all phonological patterns are subregular (belonging to a computational class weaker than the regular languages) and that phonological grammars should be limited to formal mechanisms whose descriptive capacities do not exceed the most restricted computational classes claimed to include all phonological patterns (Heinz 2010, 2018; Graf 2022).[4]

Subregular phonologists claim that these relatively weak formal mechanisms are more accurate than alternative approaches in predicting the attested typology of phonological patterns. In particular, they claim that the relatively week formal mechanisms are better than alternative approaches at excluding unattested patterns. In support of this claim, however, they focus solely on patterns that lie *outside* the subregular classes, emphasizing the ability of subregular phonology to exclude such patterns. They give no significant consideration to the patterns that fall *inside* the subregular classes, the patterns that subregular phonology does not exclude. Of course, in assessing the typological accuracy of any approach to phonology, the patterns that fall

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Rogers and Lambert 2015 employ an alphabet with a still insufficient 15 symbols. An adequate alphabet might contain twice as many symbols or more.

[3] At that point, the reader should also keep in mind that the figure of 39,916,800 merely represents the number of phonologically unique new patterns resulting from a *single* seed pattern. Using any of the patterns in Heinz's database, or the distributions of any Strictly Local language for that matter, as the seed pattern would result in its own set of distinct patterns.

[4] To be a bit more precise, Subregular Phonology adopts the Subregular Hypothesis (Heinz 2010, 2018; Graf 2022). As Heinz (2018) states, "The Subregular Hypothesis refers to the idea that phonological patterns belong to small, well-defined regions of regular stringsets and maps." Different instantiations of the hypothesis may specify the relevant regions: Strictly Local, Strictly Piecewise, Tier-Based Strictly Local, etc. They may also refer specifically to stringsets, maps, grammars, constraints, etc.

within the subregular classes are just as significant as those that lie outside the subregular classes, especially to the extent that they deviate substantially in nature from attested patterns. In presenting the results of the examination outlined above, we demonstrate that the range of patterns that fall within the subregular classes greatly exceeds the attested range.

In the remainder of this section, we first introduce the key characteristics of strictly local grammars. While we will argue that the method presented below can be applied to subregular grammars generally, we focus on strictly local grammars for two reasons. First, the strictly local class plays a central role in existing work in subregular phonology, where it appears to serve as one of the relevant "lower bounds" on computational complexity. Second, our examination takes stress patterns as the primary example, and the majority of stress patterns can be characterized as strictly local (Edlefsen et al. 2008, Rogers et al 2013, Rogers and Lambert 2019).[5] After introducing strictly local grammars, we then introduce the concepts of *derangement* and *partial derangement* of a set. Finally, we outline the method we have developed for exploring the predictions of subregular phonology and apply it to the Maranungku stress pattern represented with a two-symbol alphabet. We will typically and informally refer to the overall method as "derangement", since the method is based on the partial derangement of an alphabet and on the transformation of grammars and languages based on the partially deranged alphabet. We will also typically and informally refer to the transformation of the grammars and languages as "derangement".

In Section 2, we apply the derangement method to the Maranungku stess pattern represented with a three-symbol alphabet. Sections 3 and 4 apply the method to the Maranungku stress pattern represented with a six-syllable alphabet and to the Wergaia stress pattern represented with the same alphabet. In Sections 5 and 6, we apply the derangement method to the Maranungku and Wergaia stress patterns represented with a twelve-symbol alphabet.

Section 7 contains a formal proof of the validity of the derangement method for strictly local grammars. Section 8 presents an interim summary and discusses some key conclusions that can be reached at that point. Section 9 formally proves the validity of the derangement method for subregular grammars generally, and Section 10 contains further conclusions and discussion.

### *1.1 Strictly Local Grammars*

We can think of a strictly local grammar (McNaughton & Papert 1971) as having three parts. The first part is an *alphabet*; the second part is a set of two *string boundary symbols*; and the third part is a set of *constraints* constructed from the alphabet and the boundary symbols.[6] Each constraint specifies a sequence of symbols that is prohibited from occurring as a substring in any of the forms of the language. The language produced by the grammar is the subset of all strings that can be constructed from the alphabet flanked by the two boundary symbols and that do not contain any of the substrings prohibited by the constraints.

Consider a grammar G with an alphabet, Σ, consisting of a stressed syllable symbol, σ́, and a stressless syllable symbol, σ. (We will use a small circle over a symbol to indicate stress when we are capturing a stressed *vs*. stressless distinction without considering the distinction

[5] Edlefsen et al. (2008) argue that of the 109 attested stress patterns cataloged by Heinz (2007, 2009), 72% are Strictly Local, while the other 28% are not.

[6] We use the term "constraint" here because it will be familiar to a wider range of readers. In the literature on subregular phonology, what we refer to as a constraint is typically referred to as a "*k*-factor", a "string literal", or a "negative literal".

between primary and secondary stress.) The grammar also contains the set of boundary symbols, B, with the left boundary symbol, ⋊, and the right boundary symbol, ⋉. Finally, it contains a set of constraints, T, containing the constraints *σ́σ́, *σσ, *⋊σ. The first constraint, *σ́σ́, prohibits substrings containing adjacent stressed syllables. The second, *σσ, prohibits substrings containing adjacent stressless syllables. The third, *⋊σ, prohibits substrings where a stressless syllable follows a boundary.

(1) $G = \langle \Sigma, B, T \rangle$
$\Sigma = \{\acute{\sigma}, \sigma\}$
$B = \{⋊, ⋉\}$
$T = \{*\acute{\sigma}\acute{\sigma}, *\sigma\sigma, *⋊\sigma\}$

The grammar in (1) produces the language whose seven shortest forms are illustrated in (2).[7] The language never has a stressless syllable following a boundary, never has adjacent stressed syllables, and never has adjacent stressless syllables. The result is the Maranungku stress pattern where stress occurs on every odd-numbered syllable from the left.

(2) Maranungku pattern
⋊σ́⋉
⋊σ́σ⋉
⋊σ́σσ́⋉
⋊σ́σσ́σ⋉
⋊σ́σσ́σσ́⋉
⋊σ́σσ́σσ́σ⋉
⋊σ́σσ́σσ́σσ́⋉

A key characteristic of strictly local grammars is that they need only scan a finite window of symbols at any given time in the process of determining whether or not a form belongs to a language. The minimum scanning window required for a particular grammar is the size of the longest string prohibited by one of its constraints. If the longest prohibited string is two symbols long, for example, like the grammar for the Maranungku stress pattern above, then the smallest scanning window that the grammar can use in determining whether or not a form belongs to the language is also two symbols long. If a grammar‘s longest prohibited string is five symbols long then the smallest window that the grammar can use is also five symbols long. The size of the smallest window that can be used with a grammar is indicated with a number called a *k*-value. The *k*-value for the Maranungku pattern in (2) is two since its longest prohibited string is two symbols long. A grammar whose longest prohibited string is five symbols long would have a *k*-value of five.

### *1.2 Derangements and Partial Derangements*

As mentioned above, a *derangement* is the complete scrambling of a set. It is a bijection on a set where no element maps to itself. A *partial derangement* scrambles a set to some specified

[7] The language would also include the empty string. We have omitted consideration of the empty string in stress patterns generally, in the interest of saving space.

degree. It is a bijection with a certain number of fixed points (elements that map to themselves). Scrambling might be complete in a partial derangement; there might be no scrambling at all; or there might be some immediate degree of scrambling. A derangement is a partial derangement with zero fixed points.

One way to think about the term *partial derangement* is that it refers to the same notion as the term *permutation* but that it takes a more fine-grained view. Its view is more fine-grained in the sense that it cares about the additional detail of fixed points. Where the term *permutation* refers to the shuffling of the members of the set, the term *partial derangement* refers to the shuffling of a set while maintaining a certain number of fixed points. Focusing on this additional detail will be enlightening in some contexts, but it does not affect the total number of ways in which a set might be scrambled. The total number of partial derangements of a set is the same as the total number of permutations. Another way to think about the difference is that partial derangement cares about the degree of scrambling where permutation does not.

To illustrate, consider a set containing the five symbols A, B, C, D, and E. The bijection where every member of this set maps to itself, (3a), is the *identity permutation*. It is also the partial derangement where five out of five elements of the set are fixed points. In the bijection in (3b), A maps to A, and B maps to B; but C maps to D, D maps to E, and E maps to C. This bijection is a permutation, but it is also a partial derangement where two of the five elements are fixed points. Finally in the bijection in (3c), A maps to B, B maps to C, C maps to D, D maps E, and E maps to A. This bijection is a permutation. It is also a partial derangement where zero of five elements are fixed points. It is also a derangement.

(3) Partial derangements

a. Five fixed points

| | | |
|---|---|---|
| A | ↔ | A |
| B | ↔ | B |
| C | ↔ | C |
| D | ↔ | D |
| E | ↔ | E |

b. Two fixed points

| | | |
|---|---|---|
| A | ↔ | A |
| B | ↔ | B |
| C | ↔ | D |
| D | ↔ | E |
| E | ↔ | C |

c. Zero fixed points

| | | |
|---|---|---|
| A | ↔ | B |
| B | ↔ | C |
| C | ↔ | D |
| D | ↔ | E |
| E | ↔ | A |

A *rencontres number* is the number of partial derangements with a certain number of fixed points for a certain number of elements. More precisely, the rencontres number $D_{n,k}$ is the number of partial derangements of $n$ elements that have exactly $k$ fixed points, for $n \geq 0$ and $0 \leq k \leq n$. The number of partial derangements of $n$ elements with $k$ fixed points can usefully be summarized in a *rencontres table*, a triangular array, like the one in (4). In (4), the horizontal axis is the number fixed points $k$, and the vertical axis is the number of elements $n$. The cell at the intersection of a given value for $n$ and a given value for $k$ holds the value of the rencontres number $D_{n,k}$. For five elements with two fixed points, then, the number of partial derangements is $D_{5,2} = 20$.

(4) Rencontres Table

| *n* \ *k* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | |
| 2 | 1 | 0 | 1 | | | | | | | |
| 3 | 2 | 3 | 0 | 1 | | | | | | |
| 4 | 9 | 8 | 6 | 0 | 1 | | | | | |
| 5 | 44 | 45 | 20 | 10 | 0 | 1 | | | | |
| 6 | 265 | 264 | 135 | 40 | 15 | 0 | 1 | | | |
| 7 | 1854 | 1855 | 924 | 315 | 70 | 21 | 0 | 1 | | |
| 8 | 14833 | 14832 | 7420 | 2464 | 630 | 112 | 28 | 0 | 1 | |
| 9 | 133496 | 133497 | 66744 | 22260 | 5544 | 1134 | 168 | 36 | 0 | 1 |

Note that the sum of the values in each row of the table equals the total number of partial derangements for *n* elements, which is *n*!. For example, for a set of four elements, there are nine partial derangements with zero fixed points, eight partial derangements with one fixed point, six partial derangements with two fixed points, zero partial derangements with three fixed points, and one partial derangement with four fixed points, for a total of 24 or 4!.

Moving forward, we will typically use the terms “partial derangement” or “derangement” in contexts where the degree of scrambling is one of the factors being considered. This will be the case when we are discussing the results of applying the derangement method. One might initially suppose that the degree of scrambling significantly affects the plausibility of a scrambled pattern. This has turned out not to be the case. Stress patterns are fragile in the sense that swapping symbols, or combinations of symbols, between just two distributions is typically sufficient to render a pattern implausible. This indicates that the content of a symbol—the symbol’s phonological substance—plays a fundamental role in determining its distribution. In contexts where we are not concerned with the degree of scrambling, we will typically use the more familiar term “permutation”.

### *1.3 Derangement of Σ = {σ̊, σ}*

Here, we examine the results of deranging the alphabet, grammar, and language discussed in Section 1.2. We begin with the alphabet. As in the discussion above, alphabets will be designated Σ. Alphabet derangements will be designated DΣ, with subscripts, if necessary to distinguish between multiple derangements. The alphabet for the Maranungku stress pattern as represented in Section 1.2 was Σ = {σ̊, σ}. There are only two partial derangements possible for a Σ with two symbols. The first, $D\Sigma_1$, has two fixed points, and the second, $D\Sigma_2$, has zero fixed points.

As (5a) illustrates, $D\Sigma_1$ is the identity partial derangement. It is the bijection where the stressed syllable symbol maps onto itself and the unstressed syllable symbol maps onto itself. As (5b) illustrates, $D\Sigma_2$ is a derangement. It is a bijection where the stressed syllable symbol maps to the unstressed syllable symbol and the unstressed syllable symbol maps to the stressed syllable symbol.

(5) Partial derangements of {σ́, σ}

a. $D\Sigma_1$: 2 fixed points

σ́ ↔ σ́
σ ↔ σ

b. $D\Sigma_2$: 0 fixed points

σ́ ↔ σ
σ ↔ σ́

Creating a deranged set of constraints, $DT_i$, means substituting the symbols in the constraints in T according to mapping $D\Sigma_i$, a partial derangement of Σ. To illustrate, each of the bijections in (5) can be used to derange the set of constraints for the Maranungku stress pattern. If we replace the symbols in the constraints for the Maranungku pattern following the bijection in (5a), then we obtain $DT_1$, as in (6a). $DT_1$ is identical to the original set of constraints. If we replace the symbols in the constraints following the bijection in (5b), then we obtain $DT_2$, as in (6b). Note that, in either case, boundary symbols simply map to themselves. Put another way, partial derangements only target symbols of the original alphabet Σ.

(6) a. $DT_1$ resulting from partial derangement $D\Sigma_1$

T = {*σ́σ́, *σσ, *⋊σ}
↕ ↕ ↕ ↕ ↕ ↕
$DT_1$ = {*σ́σ́, *σσ, *⋊σ}

b. $DT_2$ resulting from partial derangement $D\Sigma_2$

T = {*σ́σ́, *σσ, *⋊σ}
↕ ↕ ↕ ↕ ↕ ↕
$DT_2$ = {*σσ, *σ́σ́, *⋊σ́}

Creating a deranged language, $DL_i$, means substituting the symbols in the strings of L according to the mapping of $D\Sigma_i$, a partial derangement of Σ. To illustrate, if we replace each symbol in each string of the Maranungku pattern according to the bijection in (5a), the result is $DL_1$, as in (7a), which is identical to the Maranungku pattern. If we replace each symbol in each string of the Maranungku pattern based on the bijection in (5b), then the result is $DL_2$, as in (7b). Note that, in either case, boundary symbols simply map to themselves. Section 7 provides the proof that a deranged grammar produces the corresponding set of deranged strings that make up the corresponding deranged language.

(7) a. $DL_1$ from partial derangement $D\Sigma_1$ b. $DL_2$ from partial derangement $D\Sigma_2$

| $DL_1$ | $DL_2$ |
|---|---|
| ⋊o̊⋉<br>↕ ↕ ↕<br>⋊o̊⋉ | ⋊o̊⋉<br>↕ ↕ ↕<br>⋊σ⋉ |
| ⋊o̊σ⋉<br>↕ ↕↕ ↕<br>⋊o̊σ⋉ | ⋊o̊σ⋉<br>↕ ↕↕ ↕<br>⋊σo̊⋉ |
| ⋊o̊σo̊⋉<br>↕ ↕↕↕ ↕<br>⋊o̊σo̊⋉ | ⋊o̊σo̊⋉<br>↕ ↕↕↕ ↕<br>⋊σo̊σ⋉ |
| ⋊o̊σo̊σ⋉<br>↕ ↕↕↕↕ ↕<br>⋊o̊σo̊σ⋉ | ⋊o̊σo̊σ⋉<br>↕ ↕↕↕↕ ↕<br>⋊σo̊σo̊⋉ |
| ⋊o̊σo̊σo̊⋉<br>↕ ↕↕↕↕↕ ↕<br>⋊o̊σo̊σo̊⋉ | ⋊o̊σo̊σo̊⋉<br>↕ ↕↕↕↕↕ ↕<br>⋊σo̊σo̊σ⋉ |
| ⋊o̊σo̊σo̊σ⋉<br>↕ ↕↕↕↕↕↕ ↕<br>⋊o̊σo̊σo̊σ⋉ | ⋊o̊σo̊σo̊σ⋉<br>↕ ↕↕↕↕↕↕ ↕<br>⋊σo̊σo̊σo̊⋉ |
| ⋊o̊σo̊σo̊σo̊⋉<br>↕ ↕↕↕↕↕↕↕ ↕<br>⋊o̊σo̊σo̊σo̊⋉ | ⋊o̊σo̊σo̊σo̊⋉<br>↕ ↕↕↕↕↕↕↕ ↕<br>⋊σo̊σo̊σo̊σ⋉ |

Since we can provide summary characterizations of languages by describing the distribution of the languages' symbols, we can also provide summary characterizations of deranged languages by describing the distribution of the deranged languages' symbols. Again using the Maranungku pattern as our example, stressed syllable symbols have the distribution in (8a) and unstressed syllable symbols have the distribution in (8b).

(8) Distribution of o̊, σ in the Maranungku pattern
   a. o̊ occurs in odd-numbered positions from the left.
   b. σ occurs in even-numbered positions from the left.

Substituting symbols in the distribution descriptions of Maranungku based on the bijection in (5a) gives us the distribution descriptions of $DL_1$.

(9) Distribution of σ̊, σ in $DL_1$

a. Distribution in L: σ̊ occurs in odd-numbered positions from the left.
↕
Distribution in $DL_1$: σ̊ occurs in odd-numbered positions from the left.

b. Distribution in L: σ occurs in even-numbered positions from the left.
↕
Distribution in $DL_1$: σ occurs in even-numbered positions from the left.

Substituting symbols in the distribution descriptions of Maranungku based on the bijection in (5b) gives us the distribution descriptions of $DL_2$.

(10) Distribution of σ̊, σ in $DL_2$

a. Distribution in L: σ̊ occurs in odd-numbered positions from the left.
↕
Distribution in $DL_2$: σ occurs in odd-numbered positions from the left.

b. Distribution in L: σ occurs in even-numbered positions from the left.
↕
Distribution in $DL_2$: σ̊ occurs in even-numbered positions from the left.

As we progress in the discussion, we will rely more and more on distribution descriptions to characterize the stress patterns of languages and deranged languages, since supplying example forms would take up an impractically large amount of space.

Since $DL_1$ in (7a) is identical to the Maranungku pattern, $DL_1$ is clearly an attested pattern. $DL_2$ in (7b) is similar to the stress pattern of Araucanian (Echeverria and Contreras 1965). The fact that the monosyllabic form of $DL_2$ is unstressed, however, ensures that this particular version of the pattern is unattested. This is true despite the fact that $DL_2$ is formally equivalent to the attested Maranungku pattern and $DL_1$. Like the Maranungku pattern and $DL_1$, $DL_2$ has a *k*-value of two. $DT_2$ is the same size as $DT_1$ and T. In fact, $DL_2$ and $DT_2$ are identical to L and T, respectively, and to $DL_1$ and $DT_1$, respectively, in every way except for the derangement of their symbols.

*2. Derangement of Σ = {σ́, σ̀, σ}*

In this section, we expand Σ from two symbols to three by replacing the generic stressed syllable symbol, σ̊, with two symbols: σ́, a symbol for a syllable with primary stress, and σ̀, a symbol for a syllable with secondary stress. This will allow the grammar to capture a three-way distinction between primary stressed syllables, secondary stressed syllables, and unstressed syllables. Given the alphabet Σ = {σ́, σ̀, σ}, the Maranungku pattern would be represented as in (11). In the pattern in (11), stress appears on every odd-numbered syllable from the left with the initial stress being primary.

(11) Maranungku pattern
⋊σ́⋉
⋊σ́σ⋉
⋊σ́σσ̀⋉
⋊σ́σσ̀σ⋉
⋊σ́σσ̀σσ̀⋉
⋊σ́σσ̀σσ̀σ⋉
⋊σ́σσ̀σσ̀σσ̀⋉

The grammar that produces the representation of the Maranungku pattern in (11) is provided in (12). Note that the longest string prohibited by a constraint has two symbols, so the language still has a *k*-value of two. With the general stressed syllable symbol, σ̊, from the previous alphabet having been replaced by a primary stressed syllable symbol, σ́, and a secondary stressed syllable symbol, σ̀, in the current alphabet, however, more constraints are required to produce the correct representation for the Maranungku pattern.

(12) G = ⟨ Σ, B, T⟩
Σ = {σ́, σ̀, σ}
B = {⋊, ⋉}
T = {*σ́σ́, *σ́σ̀, *σ̀σ́, *σ̀σ̀, *σσ́, *σσ, *⋊σ̀, *⋊σ}

The distribution of the individual symbols resulting from the grammar in (12) is given in (13). The primary stressed syllable symbol occurs in initial position. The secondary stressed syllable symbol occurs in non-initial odd-numbered positions. The unstressed syllable symbol occurs in even-numbered positions.

(13) Distribution of σ́, σ̀, σ in the Maranungku pattern
a. σ́ occurs in initial position.
b. σ̀ occurs in non-initial odd-numbered positions from the left.
c. σ occurs in even-numbered positions from the left.

There are six partial derangements for the seed Σ in (13): one with three fixed points, three with one fixed point, and two with zero fixed points. Partially deranging the grammar and strings of the Maranungku pattern based on the six partially deranged DΣ yields six distinct stress patterns, each of which is summarized below. Note that from this point forward we will list symbols in $D\Sigma_i$ in an order that reflects their partially deranged mapping. The symbol in the first position in Σ maps to the symbol in the first position in $D\Sigma_i$. The symbol in the second position in Σ maps to the symbol in the second position in $D\Sigma_i$, and so on. This will allow us to more easily and conveniently distinguish between the partial derangements.

Since the partial arrangement with three fixed points, $D\Sigma_1$, is the identity partial derangement, and the resulting grammar and language are identical in every way to the seed pattern, we omit illustration of $D\Sigma_1$ and begin with the partial derangements that have a single fixed point. The first, $D\Sigma_2$, is the partial derangement where the primary stressed syllable symbol is the fixed point and the secondary stressed syllable symbol and the unstressed syllable symbol are swapped. As (14) illustrates, the resulting pattern retains primary stress on initial syllables, but secondary stress now appears on every even-numbered syllable from the left. The new

distribution of secondary stress produces a clash at the left edge in both even- and odd-parity forms. This partial derangement is the most plausible of the non-identity partial derangements, but it is unattested. As Hyde (2002, 2016, 2025) points out, the locations of clashes and lapses in quantity-insensitive systems are tightly restricted. Though there are attested patterns where clash occurs at the left edge in odd-parity forms, there are no attested patterns where clash occurs in both even-parity forms and odd-parity forms, in initial position or in any other position.

(14) $DL_2$
⋊σ́⋉
⋊σ́σ̀⋉
⋊σ́σ̀σ⋉
⋊σ́σ̀σσ̀⋉
⋊σ́σ̀σσ̀σ⋉
⋊σ́σ̀σσ̀σσ̀⋉
⋊σ́σ̀σσ̀σσ̀σ⋉

As in T, the longest prohibited string in $DT_2$ has two symbols, so $DL_2$ has a *k*-value of two. The number of constraints in the two sets is also the same.

(15) $DG_2 = \langle D\Sigma_2, B, DT_2 \rangle$
$D\Sigma_2 = \{\acute{\sigma}, \sigma, \grave{\sigma}\}$
$B = \{\rtimes, \ltimes\}$
$DT_2 = \{*\acute{\sigma}\acute{\sigma}, *\acute{\sigma}\sigma, *\sigma\acute{\sigma}, *\sigma\sigma, *\grave{\sigma}\acute{\sigma}, *\grave{\sigma}\grave{\sigma}, *\rtimes\sigma, *\rtimes\grave{\sigma}\}$

The distributions in $DL_2$ are the same distributions that occur in the Maranungku pattern, but the symbols that occur in second and third distributions have been swapped.

(16) Distribution of $\acute{\sigma}$, $\grave{\sigma}$, σ in $DL_2$
a. $\acute{\sigma}$ occurs in initial position.
b. σ occurs in non-initial odd-numbered positions from the left.
c. $\grave{\sigma}$ occurs in even-numbered positions from the left.

The next partial derangement, $DL_3$, holds the secondary stressed syllable symbol as a fixed point and swaps the primary stress syllable symbol and the unstressed syllable symbol. As (17) illustrates, an unstressed syllable occurs in initial position, secondary stressed syllables occur in non-initial odd-numbered positions, and primary stressed syllables occur in even-numbered positions. The result is an unbroken string of stressed syllables, after an initial stressless syllable, alternating between primary stress and secondary stress. The minimal form has no stress at all. The pattern deviates from the characteristic properties of attested stress systems to such a degree that it is simply implausible.

(17) $DL_3$
⋊σ⋉
⋊σσ́⋉
⋊σσ́σ̀⋉
⋊σσ́σ̀σ́⋉
⋊σσ́σ̀σ́σ̀⋉
⋊σσ́σ̀σ́σ̀σ́⋉
⋊σσ́σ̀σ́σ̀σ́σ̀⋉

As in T, the longest string prohibited by a constraint in $DT_3$ has two symbols, so $DL_3$ has a *k*-value of two. The number of constraints in the two sets is also the same.

(18) $DG_3 = \langle D\Sigma_3, B, DT_3 \rangle$
$D\Sigma_3 = \{\sigma, \grave{\sigma}, \acute{\sigma}\}$
$B = \{⋊, ⋉\}$
$DT_3 = \{*\sigma\sigma, *\sigma\grave{\sigma}, *\grave{\sigma}\sigma, *\grave{\sigma}\grave{\sigma}, *\acute{\sigma}\sigma, *\acute{\sigma}\acute{\sigma}, *⋊\grave{\sigma}, *⋊\acute{\sigma}\}$

The distributions in $DL_3$ are the same distributions that occur in the seed pattern, but the symbols that occur in first and third distributions have been swapped.

(19) Distribution of $\acute{\sigma}$, $\grave{\sigma}$, $\sigma$ in $DL_3$
a. $\sigma$ occurs in initial position.
b. $\grave{\sigma}$ occurs in non-initial odd-numbered positions from the left.
c. $\acute{\sigma}$ occurs in even-numbered positions from the left.

In the next partial derangement, $DL_4$, the unstressed syllable symbol is a fixed point and the primary stressed syllable symbol and the secondary stressed syllable symbol are swapped. The resulting pattern is one where the initial syllable has secondary stress and non-initial odd-numbered syllables have primary stress. Note that the single syllable of the minimal form has a secondary stress rather than a primary stress. The pattern is implausible.

(20) $DL_4$
⋊σ̀⋉
⋊σ̀σ⋉
⋊σ̀σσ́⋉
⋊σ̀σσ́σ⋉
⋊σ̀σσ́σσ́⋉
⋊σ̀σσ́σσ́σ⋉
⋊σ̀σσ́σσ́σσ́⋉

As in T, the longest string prohibited by a constraint in $DT_4$ has two symbols, so $DL_4$ has a *k*-value of two. The number of constraints in the two sets is also the same.

(21) $DG_4 = \langle D\Sigma_4, B, DT_4 \rangle$
$D\Sigma_4 = \{\grave{\sigma}, \acute{\sigma}, \sigma\}$
$B = \{\rtimes, \ltimes\}$
$DT_4 = \{*\grave{\sigma}\grave{\sigma}, *\grave{\sigma}\acute{\sigma}, *\acute{\sigma}\grave{\sigma}, *\acute{\sigma}\acute{\sigma}, *\sigma\grave{\sigma}, *\sigma\sigma, *\rtimes\acute{\sigma}, *\rtimes\sigma\}$

The distributions in $DL_4$ are the same distributions that occur in the Maranungku pattern, but the symbols that occur in first and second distributions have been swapped.

(22) Distribution of $\acute{\sigma}$, $\grave{\sigma}$, $\sigma$ in $DL_4$
a. $\grave{\sigma}$ occurs in initial position.
b. $\acute{\sigma}$ occurs in non-initial odd-numbered positions from the left.
c. $\sigma$ occurs in even-numbered positions from the left.

The final two patterns are partial derangements with zero fixed points. In the first, $DL_5$, the seed pattern's primary stressed syllable symbol maps to the unstressed syllable symbol, the seed pattern's secondary stressed syllable symbol maps to the primary stressed syllable symbol, and the seed pattern's unstressed syllable symbol maps to the secondary stressed syllable symbol. In this language, there is an initial secondary stress, then, with primary stresses on odd-numbered syllables. The multiple primary stresses in longer forms, the lack of a primary stress in the minimal form, and the clash at the left edge in all non-minimal forms all make the pattern implausible.

(23) $DL_5$
$\rtimes\grave{\sigma}\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\sigma\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\sigma\acute{\sigma}\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\sigma\acute{\sigma}\sigma\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\sigma\acute{\sigma}\sigma\acute{\sigma}\ltimes$
$\rtimes\grave{\sigma}\acute{\sigma}\sigma\acute{\sigma}\sigma\acute{\sigma}\sigma\ltimes$

As in T, the longest string prohibited by a constraint in $DT_5$ has two symbols, so $DL_5$ has a *k* value of two. The number of constraints in the two sets is also the same.

(24) $DG_5 = \langle D\Sigma_5, B, DT_5 \rangle$
$D\Sigma_5 = \{\grave{\sigma}, \sigma, \acute{\sigma}\}$
$B = \{\rtimes, \ltimes\}$
$DT_5 = \{*\grave{\sigma}\grave{\sigma}, *\grave{\sigma}\sigma, *\sigma\grave{\sigma}, *\sigma\sigma, *\acute{\sigma}\grave{\sigma}, *\acute{\sigma}\acute{\sigma}, *\rtimes\sigma, *\rtimes\acute{\sigma}\}$

The distributions in $DL_5$ are the same distributions that occur in the Maranungku pattern, but the symbols have been completely deranged so that none occurs in the same distribution as in the Maranungku pattern.

(25) Distribution of σ́, σ̀, σ in $DL_5$
   a. σ̀ occurs in initial position.
   b. σ occurs in non-initial odd-numbered positions from the left.
   c. σ́ occurs in even-numbered positions from the left.

In the final partially deranged pattern, $DL_6$, the seed pattern's primary stressed syllable symbol maps to the secondary stressed syllable symbol; the seed pattern's secondary stressed syllable symbol maps to the unstressed syllable symbol, and the seed pattern's unstressed syllable symbol maps to the primary stressed syllable symbol. As (26) illustrates, primary stress in $DL_6$ appears on non-initial odd-numbered syllables from the left and secondary stress appears on even-numbered syllables from the left. The result is that, after an initial unstressed syllable, there is a string of alternating secondary and primary stresses through the end of the word. The minimal form has no stress at all, and the two-syllable form has only a secondary stress. The result is implausible.

(26) $DL_6$
⋊σ⋉
⋊σσ̀⋉
⋊σσ̀σ́⋉
⋊σσ̀σ́σ̀⋉
⋊σσ̀σ́σ̀σ́⋉
⋊σσ̀σ́σ̀σ́σ̀⋉
⋊σσ̀σ́σ̀σ́σ̀σ́⋉

As in T, the longest string prohibited by a constraint in $DT_6$ has two symbols, so $DL_6$ has a *k*-value of two. The number of constraints in the two sets is also the same.

(27) $DG_6 = \langle D\Sigma_6, B, DT_6 \rangle$
$D\Sigma_6 = \{\sigma, \acute{\sigma}, \grave{\sigma}\}$
$B = \{⋊, ⋉\}$
$DT_6 = \{*\sigma\sigma, *\sigma\acute{\sigma}, *\acute{\sigma}\sigma, *\acute{\sigma}\acute{\sigma}, *\grave{\sigma}\sigma, *\grave{\sigma}\grave{\sigma}, *⋊\acute{\sigma}, *⋊\grave{\sigma}\}$

The distributions in $DL_6$ are the same distributions that occur in the seed pattern, but the symbols have been completely scrambled so that none occurs in its original distribution.

(28) Distribution of σ́, σ̀, σ in $DL_6$
   a. σ occurs in initial position.
   b. σ́ occurs in non-initial odd-numbered positions from the left.
   c. σ̀ occurs in even-numbered positions from the left.

In summary, there are six partial derangements of $\Sigma = \{\acute{\sigma}, \grave{\sigma}, \sigma\}$. Derangement of Maranungku stress pattern results in six distinct stress patterns. With the exception of the identity partial derangement, none is attested. Of the five unattested partially deranged patterns, only one is somewhat plausible, and it does not match what we currently understand about the typological distribution of clashes. The remaining four partially deranged patterns are implausible.

We might have anticipated that a greater number of fixed points would result in more plausible patterns. This turns out not to be the case, at least with this set of partial derangements. Two of three partial derangements with one fixed point yield implausible results. Both of the partial derangements with zero fixed points yield implausible results. The plausibility of a result seems more closely related to the content of the symbols—to their substance—and the particular distributions that they are assigned to. The factor that seems to result in the most plausible patterns in this context is having Σ's primary stressed syllable symbol, in particular, as a fixed point. Since typical distributions are so different, swapping the primary stressed syllable symbol and secondary stressed syllable symbol, or swapping the primary stressed syllable symbol and unstressed syllable symbol, results in patterns that are implausible.

*3. Heavy and light syllables: Maranungku*

In this section, we expand the alphabet from three symbols to six, distinguishing between heavy and light syllables for each degree of stress. The primary stressed syllable symbol, σ́, is split into a heavy version and a light version, H́ and Ĺ, respectively. The secondary stressed syllable symbol, σ̀, is split into a heavy version and a light version, H̀ and L̀, respectively. Finally, the unstressed syllable symbol, σ, is split into a heavy version and a light version, H and L respectively. An expanded alphabet like Σ = {H́, Ĺ, H̀, L̀, H, L} takes a small step towards what would be adequate in any serious investigation of stress. It allows the analysis to accommodate the minimal, most generic syllable weight distinctions that stress is sensitive to in a quantity-sensitive pattern, and it allows for comparison between quantity-sensitive patterns and quantity-insensitive patterns.

Focusing again on Maranungku as the seed pattern, we begin by illustrating how the Maranungku pattern is represented with the expanded alphabet. We then examine the results of derangement of the Maranungku pattern given the expanded alphabet. The results can usefully be classified using three characteristics: *quantity insensitivity*, *quantity sensitivity*, and *quantity restrictedness*. In preparation for the discussion of the derangement results, it will be helpful to clarify how we will be using each of these terms in this section.

We will employ the term *quantity sensitivity* under a very general meaning. Quantity sensitivity will mean not only the preference for greater degrees of stress to occur with heavier syllables and lesser degrees of stress to occur with lighter syllables, a situation frequently found among the world's languages, but it will also encompass the reverse situation where a greater degree of stress is preferred with lighter syllables and a lesser degree of stress is preferred with heavier syllables, a situation not found among the world's languages. The patterns of quantity sensitivity in this section also includes a range of distributions of quantity-based contrasts that differs from those seen in attested languages.

The term *quantity restricted* will indicate the situation where a particular distribution allows only one type of syllable weight. This circumstance is reminiscent of some phenomena found among the world's languages, but the particular manner in which we find it in the deranged patterns below is not reminiscent of attested phenomena. When a particular distribution only allows a particular syllable weight in the patterns in this section, it allows syllables with that weight to occur in free variation with two different degrees of stress. For example, a distribution may only allow heavy syllables, but the heavy syllables might freely vary between primary stress and secondary stress.

We will use the term *quantity insensitive* to mean neither quantity sensitive nor quantity restricted. A quantity-insensitive distribution is one that completely ignores distinctions in syllable weight.

When represented with the expanded Σ, as in (29), the Maranungku stress pattern is essentially that same pattern that we saw in the previous section. Syllables with primary stress, whether heavy or light, occur in initial position. Syllables with secondary stress, whether heavy or light, occur in non-initial odd-numbered positions. Syllables without stress, whether heavy or light, occur in even-numbered positions.

(29) Maranungku pattern

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊H́⋉ | ⋊H́HH̀H⋉ | ⋊H́HH̀HH̀⋉ | ⋊Ĺ H̀HH̀⋉ |
| ⋊Ĺ⋉ | ⋊H́HH̀L⋉ | ⋊H́HH̀HL̀⋉ | ⋊Ĺ H̀HL̀⋉ |
| | ⋊H́HL̀H⋉ | ⋊H́HH̀L H̀⋉ | ⋊Ĺ H̀L H̀⋉ |
| *2 syllables* | ⋊H́HL̀L⋉ | ⋊H́HH̀L L̀⋉ | ⋊Ĺ H̀L L̀⋉ |
| ⋊H́H⋉ | ⋊H́L H̀H⋉ | ⋊H́HL̀H H̀⋉ | ⋊Ĺ HL̀H H̀⋉ |
| ⋊H́L⋉ | ⋊H́L H̀L⋉ | ⋊H́HL̀HL̀⋉ | ⋊Ĺ HL̀HL̀⋉ |
| ⋊Ĺ H⋉ | ⋊H́L L̀H⋉ | ⋊H́HL̀L H̀⋉ | ⋊Ĺ HL̀L H̀⋉ |
| ⋊Ĺ L⋉ | ⋊H́L L̀L⋉ | ⋊H́HL̀L L̀⋉ | ⋊Ĺ HL̀L L̀⋉ |
| | ⋊Ĺ H̀HH̀⋉ | ⋊H́L H̀H H̀⋉ | ⋊Ĺ L H̀H H̀⋉ |
| 3 syllables | ⋊Ĺ H̀HL̀⋉ | ⋊H́L H̀HL̀⋉ | ⋊Ĺ L H̀HL̀⋉ |
| ⋊H́HH̀⋉ | ⋊Ĺ HL̀H⋉ | ⋊H́L H̀L H̀⋉ | ⋊Ĺ L H̀L H̀⋉ |
| ⋊H́HL̀⋉ | ⋊Ĺ HL̀L⋉ | ⋊H́L H̀L L̀⋉ | ⋊Ĺ L H̀L L̀⋉ |
| ⋊H́L H̀⋉ | ⋊Ĺ L H̀H⋉ | ⋊H́L L̀H H̀⋉ | ⋊Ĺ L L̀H H̀⋉ |
| ⋊H́L L̀⋉ | ⋊Ĺ L H̀L⋉ | ⋊H́L L̀H L̀⋉ | ⋊Ĺ L L̀H L̀⋉ |
| ⋊Ĺ H H̀⋉ | ⋊Ĺ L L̀H⋉ | ⋊H́L L̀L H̀⋉ | ⋊Ĺ L L̀L H̀⋉ |
| ⋊Ĺ H L̀⋉ | ⋊Ĺ L L̀L⋉ | ⋊H́L L̀L L̀⋉ | ⋊Ĺ L L̀L L̀⋉ |
| ⋊Ĺ L H̀⋉ | | | |
| ⋊Ĺ L L̀⋉ | | | |

The grammar in (30) is responsible for the pattern in (29). Notice that the expansion of the alphabet in this case does not affect the Maranungku pattern's *k*-value. Though the size of T has nearly quadrupled to capture the symbol combinations necessary to produce the pattern under the expanded alphabet, the longest string prohibited by a constraint still has two characters, so the pattern still has a *k*-value of two.

(30) G = ⟨Σ, B, T⟩
Σ = {H́, Ĺ, H̀, L̀, H, L}
B = {⋊, ⋉}
T = {*H́H́, *H́Ĺ, *H́H̀, *H́L̀, *ĹH́, *ĹĹ, *ĹH̀, *ĹL̀, *H̀H́, *H̀Ĺ, *H̀H̀, *H̀L̀, *L̀H́, *L̀Ĺ, *L̀H̀, *L̀L̀, *HH́, *HĹ, *HH, *HL, *LH́, *LĹ, *LH, *LL, *⋊H̀, *⋊L̀, *⋊H, *⋊L}

Notice also that the distributions for the Maranungku pattern are the same distributions that we saw for the Maranungku pattern in section 3. The difference in (31) is that two characters, rather than a single character, are assigned to each distribution.

(31) Distribution of H́, Ĺ, H̀, L̀, H, L in the Maranungku pattern
   a. H́, Ĺ occur in initial position.
   b. H̀, L̀ occur in non-initial odd-numbered positions from the left.
   c. H, L occur in even-numbered positions from the left.

In examining some of the distinct patterns that arise from deranging the Maranungku stress pattern with the expanded alphabet, we will encounter a situation for the first time where the distinct patterns resulting from derangement are fewer than the possible partial derangements. This is a good place, then, to pause and explain how the distribution assignments help us to calculate the number of distinct patterns.

A grammar has the effect of assigning each symbol in the alphabet to a particular distribution in the strings of the language. In the examples examined in previous sections, only one symbol was assigned to any given distribution. When only one symbol is assigned to any given distribution, the number of distinct patterns produced from a seed pattern is equal to the number of possible partial derangements. In the current example, however, multiple symbols are assigned to each distribution. Anytime multiple symbols are assigned to the same distribution, there will be fewer distinct patterns than possible partial derangements.[8]

It is a simple matter to calculate how many distinct patterns will result from the derangement of any given seed pattern. Let $\delta_1$, $\delta_2$, $\delta_3$, …, $\delta_n$ be the distributions of a seed pattern that symbols are assigned to, let $|\delta_i|$ be the number of symbols assigned to $\delta_i$, and let $|\Sigma|$ be the number of symbols in Σ. Then the number of distinct patterns resulting from derangement of the seed pattern is

$$\frac{|\Sigma|!}{|\delta_1|! \cdot |\delta_2|! \cdot |\delta_3|! \cdot \ldots \cdot |\delta_n|!}$$

Given the current alphabet, Σ = {H́, Ĺ, H̀, L̀, H, L}, and the distributions for the Maranungku seed pattern provided in (31), the number of distinct patterns resulting from derangement in the current example is

$$\frac{6!}{2! \cdot 2! \cdot 2!} = 90$$

We turn now the patterns that result from derangement of the Maranungku pattern given Σ = {H́, Ĺ, H̀, L̀, H, L}. The patterns resulting from derangement can usefully be divided into five types: patterns where every distribution is quantity insensitive, patterns where every distribution is quantity sensitive, patterns where two distributions are quantity sensitive and one quantity insensitive, patterns where two distributions are quantity restricted and one quantity

[8] To this point, we have defined a symbol's distribution in terms of the positions that it can occupy in a string, and we will continue to do so in the remainder of the paper. Defining distributions in terms of string positions is the traditional way that quantity-insensitive stress patterns are described, and it is a particularly transparent method for quantity-insensitive patterns. It also happens to be a particularly transparent method for patterns resulting from the derangement of quantity-insensitive patterns. It is important to note, however, that there are alternative methods, though we will not employ them here.

insensitive, and patterns where two distributions are quantity restricted and one is quantity sensitive. In the interest of space, we consider only one representative example of each type in detail.

We consider the thoroughly quantity-insensitive deranged patterns first. When derangement assigns pairs of syllables that differ in weight but that have the same degree of stress to the same distribution, the result is that stress assignment is quantity insensitive within that distribution. When this situation holds for every distribution within a language, the result is that the language's overall stress pattern is completely quantity insensitive. Of the 90 distinct patterns resulting from derangement of the Maranungku stress pattern, six are completely quantity insensitive. Unsurprisingly, the six quantity-insensitive patterns are the same six quantity-insensitive patterns that we saw emerging as the result of the derangement of Σ = {σ́, σ̀, σ} in Section 2. Of the six distinct quantity-insensitive patterns (resulting from 48 partial derangements with the current alphabet), only the pattern that is identical to the Maranungku pattern is actually attested. The remaining five patterns are unattested, and four of these are implausible.

One of these completely quantity-insensitive patterns results from $DL_1$ (and seven other derangements). $DL_1$ is a partial derangement that has no fixed points. The resulting pattern is one where initial syllables are stressless, whether heavy or light, non-initial odd-numbered syllables have primary stress, whether heavy or light, and even-numbered syllables have secondary stress, whether heavy or light. After an initial unstressed syllable, then, each form has a string of alternating secondary and primary stresses through the end of the word. One-syllable forms have no stress at all, and two syllable forms have only secondary stress. In essence, this derangement replicates the derangement from Section 2 illustrated in (26-28). Like that previous example, it is implausible.

(32) $DL_1$ example forms

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊H⋉ | ⋊HÀÁHÀ⋉ | ⋊HÀÁHÀÁ⋉ | ⋊LÀÁHÀÁ⋉ |
| ⋊L⋉ | ⋊HÀÁHĹ⋉ | ⋊HÀÁHÀĹ⋉ | ⋊LÀÁHÀĹ⋉ |
| | ⋊HÀĹLÀ⋉ | ⋊HÀÁĹLÁ⋉ | ⋊LÀÁĹLÁ⋉ |
| *2 syllables* | ⋊HÀĹĹL⋉ | ⋊HÀÁĹĹĹ⋉ | ⋊LÀÁĹĹĹ⋉ |
| ⋊HÀ⋉ | ⋊HĹÁHÀ⋉ | ⋊HÀĹLÀÁ⋉ | ⋊LÀĹLÀÁ⋉ |
| ⋊HĹ⋉ | ⋊HĹÁHĹ⋉ | ⋊HÀĹLÀĹ⋉ | ⋊LÀĹLÀĹ⋉ |
| ⋊LÀ⋉ | ⋊HĹĹLÁ⋉ | ⋊HÀĹĹLÁ⋉ | ⋊LÀĹĹLÁ⋉ |
| ⋊LĹ⋉ | ⋊HĹĹĹL⋉ | ⋊HÀĹĹĹĹ⋉ | ⋊LÀĹĹĹĹ⋉ |
| | ⋊LÀÁHÀ⋉ | ⋊HĹÁHÀÁ⋉ | ⋊LĹÁHÀÁ⋉ |
| 3 syllables | ⋊LÀÁHĹ⋉ | ⋊HĹÁHÀĹ⋉ | ⋊LĹÁHÀĹ⋉ |
| ⋊HÀÁ⋉ | ⋊LÀĹLÀ⋉ | ⋊HĹÁHĹLÁ⋉ | ⋊LĹÁHĹLÁ⋉ |
| ⋊HÀĹ⋉ | ⋊LÀĹĹL⋉ | ⋊HĹÁHĹĹĹ⋉ | ⋊LĹÁHĹĹĹ⋉ |
| ⋊HĹÁ⋉ | ⋊LĹÁHÀ⋉ | ⋊HĹĹLÀÁ⋉ | ⋊LĹĹLÀÁ⋉ |
| ⋊HĹĹ⋉ | ⋊LĹÁHĹ⋉ | ⋊HĹĹLÀĹ⋉ | ⋊LĹĹLÀĹ⋉ |
| ⋊LÀÁ⋉ | ⋊LĹĹLÁ⋉ | ⋊HĹĹĹLÁ⋉ | ⋊LĹĹĹLÁ⋉ |
| ⋊LÀĹ⋉ | ⋊LĹĹĹĹ⋉ | ⋊HĹĹĹĹĹ⋉ | ⋊LĹĹĹĹĹ⋉ |
| ⋊LĹÁ⋉ | | | |
| ⋊LĹĹ⋉ | | | |

The grammar, $DG_1$, that produces $DL_1$ is given in (33). Note that $DT_1$ is the same size as T of the seed pattern. As in T, the longest string prohibited by a constraint in $DT_1$ has two symbols, so $DL_1$ has a *k*-value of two.

(33) $DG_1 = \langle D\Sigma_1, B, DT_1 \rangle$
$D\Sigma_1 = \{H, L, \acute{H}, \acute{L}, \grave{H}, \grave{L}\}$
$B = \{\rtimes, \ltimes\}$
$DT_1 = \{*HH, *HL, *H\acute{H}, *H\acute{L}, *LH, *LL, *L\acute{H}, *L\acute{L}, *\acute{H}H, *\acute{H}L, *\acute{H}\acute{H}, *\acute{H}\acute{L}, *\acute{L}H, *\acute{L}L, *\acute{L}\acute{H}, *\acute{L}\acute{L}, *\grave{H}H, *\grave{H}L, *\grave{H}\grave{H}, *\grave{H}\grave{L}, *\grave{L}H, *\grave{L}L, *\grave{L}\grave{H}, *\grave{L}\grave{L}, *\rtimes\acute{H}, *\rtimes\acute{L}, *\rtimes\grave{H}, *\rtimes\grave{L}\}$

As (34) indicates, the derangement results in H and L being assigned to initial position, $\acute{H}$ and $\acute{L}$ to non-initial odd-numbered positions, and $\grave{H}$ and $\grave{L}$ to even-numbered positions. In effect, then, the derangement shifts H, L to the distribution occupied by $\acute{H}$, $\acute{L}$ in the seed pattern, shifts $\acute{H}$, $\acute{L}$ to the distribution occupied by $\grave{H}$, $\grave{L}$ in the seed pattern, and shifts $\grave{H}$, $\grave{L}$ to the distribution occupied by H, L in the seed pattern. Since the derangement still results in a pair consisting of a light syllable and a heavy syllable with the same degree of stress assigned to each distribution, the result is still quantity insensitivity in each distribution and quantity insensitivity overall.

(34) Distribution of $\acute{H}$, $\acute{L}$, $\grave{H}$, $\grave{L}$, H, L in $DL_1$
a. H, L occur in initial position.
b. $\acute{H}$, $\acute{L}$ occur in non-initial odd-numbered positions from the left.
c. $\grave{H}$, $\grave{L}$ occur in even-numbered positions from the left.

Next, we consider situations where the pattern is thoroughly quantity sensitive. When derangement results in a pair of symbols being assigned to a particular distribution where one member of the pair is heavy and the other light and where the two do not match in their degree of stress, the result is that the level of stress is quantity sensitive in that distribution. When this situation holds for every distribution in the language, the language is completely quantity sensitive. Of the 90 distinct patterns resulting from derangement of the Maranungku seed pattern, twelve are thoroughly quantity sensitive. All twelve patterns are unattested. Given the nature of the quantity sensitivity and the restriction of particular types of quantity sensitivity to individual distributions, these patterns are also implausible.

One of the twelve completely quantity-sensitive patterns results from $DL_2$ (and seven other derangements). $DL_2$ is a partial derangement that has zero fixed points. In the resulting pattern, initial syllables carry secondary stress if heavy, but they are stressless if light. Non-initial odd-numbered syllables are stressless if heavy, but they carry primary stress if light. Even-numbered syllables have primary stress if heavy, but they carry secondary stress if light.

(35) $DL_2$ example forms

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊H̀⋉ | ⋊H̀ḾĹḾ⋉ | ⋊H̀ḾĹḾĹ⋉ | ⋊LḾĹḾĹ⋉ |
| ⋊L⋉ | ⋊H̀ḾĹL̀⋉ | ⋊H̀ḾĹḾH⋉ | ⋊LḾĹḾH⋉ |
| | ⋊H̀ḾḾḾ⋉ | ⋊H̀ḾĹL̀Ĺ⋉ | ⋊LḾĹL̀Ĺ⋉ |
| *2 syllables* | ⋊H̀ḾḾL̀⋉ | ⋊H̀ḾĹL̀H⋉ | ⋊LḾĹL̀H⋉ |
| ⋊H̀Ḿ⋉ | ⋊H̀L̀ĹḾ⋉ | ⋊H̀ḾḾḾĹ⋉ | ⋊LḾḾḾĹ⋉ |
| ⋊H̀L̀⋉ | ⋊H̀L̀ĹL̀⋉ | ⋊H̀ḾḾḾH⋉ | ⋊LḾḾḾH⋉ |
| ⋊LḾ⋉ | ⋊H̀L̀HḾ⋉ | ⋊H̀ḾḾL̀Ĺ⋉ | ⋊LḾḾL̀Ĺ⋉ |
| ⋊LL̀⋉ | ⋊H̀L̀HL̀⋉ | ⋊H̀ḾḾL̀H⋉ | ⋊LḾḾL̀H⋉ |
| | ⋊LḾĹḾ⋉ | ⋊H̀L̀ĹḾĹ⋉ | ⋊LL̀ĹḾĹ⋉ |
| 3 syllables | ⋊LḾĹL̀⋉ | ⋊H̀L̀ĹḾH⋉ | ⋊LL̀ĹḾH⋉ |
| ⋊H̀ḾĹ⋉ | ⋊LḾḾḾ⋉ | ⋊H̀L̀Ĺ L̀Ĺ⋉ | ⋊LL̀ĹL̀Ĺ⋉ |
| ⋊H̀ḾH⋉ | ⋊LḾḾL̀⋉ | ⋊H̀L̀ĹL̀H⋉ | ⋊LL̀ĹL̀H⋉ |
| ⋊H̀L̀Ĺ⋉ | ⋊LL̀ĹḾ⋉ | ⋊H̀L̀ḾḾĹ⋉ | ⋊LL̀HḾĹ⋉ |
| ⋊H̀L̀H⋉ | ⋊LL̀ĹL̀⋉ | ⋊H̀L̀HḾH⋉ | ⋊LL̀HḾH⋉ |
| ⋊LḾĹ⋉ | ⋊LL̀HḾ⋉ | ⋊H̀L̀HL̀Ĺ⋉ | ⋊LL̀HL̀Ĺ⋉ |
| ⋊LḾH⋉ | ⋊LL̀HL̀⋉ | ⋊H̀L̀HL̀H⋉ | ⋊LL̀HL̀H⋉ |
| ⋊LL̀Ĺ⋉ | | | |
| ⋊LL̀H⋉ | | | |

The grammar, $DG_2$, that produces $DL_2$ is given in (36). Note that $DT_2$ is the same size as T of the seed pattern. As in T, the longest string prohibited by a constraint in $DT_2$ has two symbols, so $DL_2$ also has a *k*-value of two.

(36) $DG_2 = \langle D\Sigma_2, B, DT_2 \rangle$
$D\Sigma_2 = \{$H̀, L, H, Ĺ, Ḿ, L̀$\}$
$B = \{\rtimes, \ltimes\}$
$DT_2 = \{$*H̀H̀, *H̀L, *H̀Ĺ, *H̀H, *LH̀, *LL, *LĹ, *LH, *ĹH̀, *ĹL, *ĹĹ, *ĹH, *HH̀, *HL, *HĹ, *HH, *ḾH̀, *ḾL, *ḾḾ, *ḾH̀, *ḾĹ, *L̀H̀, *L̀L, *L̀Ḿ, *L̀L̀, *⋊Ĺ, *⋊H, *⋊Ḿ, *⋊L̀$\}$

As (37) indicates, the derangement that produces $DL_2$ results in H̀ and L being assigned to initial position, H and Ĺ to non-initial odd-numbered positions, and Ḿ and L̀ to even-numbered positions. Since the derangement results in a pair consisting of a light syllable and a heavy syllable with different degrees of stress assigned to each distribution, the result is quantity sensitivity in each distribution and quantity sensitivity overall.

(37) Distribution of Ḿ, Ĺ, H̀, L̀, H, L in the $DL_2$ pattern
a. H̀, L occur in initial position.
b. H, Ĺ occur in non-initial odd-numbered positions from the left.
c. Ḿ, L̀ occur in even-numbered positions from the left.

Next, we consider patterns that are partially quantity sensitive and partially quantity insensitive. Of the 90 distinct patterns resulting from the derangement of the Maranungku seed pattern, 18 are quantity sensitive in two distributions and quantity insensitive in one distribution. All 18 patterns are unattested. Given the nature of the quantity sensitivity and the restriction of

particular types of quantity sensitivity to individual distributions, the patterns are also implausible.

One of the 18 mixed quantity-sensitive and quantity-insensitive patterns results from the derangement that produces $DL_3$ (and seven other derangements). $DL_3$ results from a partial derangement with four fixed points. Only the secondary stressed light syllable symbol and the unstressed light syllable symbol have been swapped from their positions in the seed pattern. Primary stress appears in initial position whether the initial syllable is heavy or light, making primary stress quantity insensitive. Secondary stress is quantity sensitive, however, and the nature of its quantity sensitivity varies by position. Non-initial odd-numbered positions have secondary stress only if the syllable is heavy; light syllables are unstressed. Even-numbered positions have secondary stress only if the syllable is light; heavy syllables are unstressed.

(38) $DL_3$ example forms

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊Ĥ⋉ | ⋊ĤL̀ȞL⋉ | ⋊ĤL̀ȞL̀Ȟ⋉ | ⋊ĹL̀ȞL̀Ȟ⋉ |
| ⋊Ĺ⋉ | ⋊ĤL̀ȞH⋉ | ⋊ĤL̀ȞL̀L⋉ | ⋊ĹL̀ȞL̀L⋉ |
| | ⋊ĤL̀L̀L⋉ | ⋊ĤL̀ȞHȞ⋉ | ⋊ĹL̀ȞHȞ⋉ |
| *2 syllables* | ⋊ĤL̀LH⋉ | ⋊ĤL̀ȞHL⋉ | ⋊ĹL̀ȞHL⋉ |
| ⋊ĤL̀⋉ | ⋊ĤHȞL⋉ | ⋊ĤL̀L̀Ȟ⋉ | ⋊ĹL̀L̀Ȟ⋉ |
| ⋊ĤH⋉ | ⋊ĤHȞH⋉ | ⋊ĤL̀L̀L⋉ | ⋊ĹL̀L̀L⋉ |
| ⋊Ĺ L̀⋉ | ⋊ĤHL̀L̀⋉ | ⋊ĤL̀LHȞ⋉ | ⋊ĹL̀LHȞ⋉ |
| ⋊ĹH⋉ | ⋊ĤHLH⋉ | ⋊ĤL̀LHL⋉ | ⋊ĹL̀LHL⋉ |
| | ⋊Ĺ L̀ȞL̀⋉ | ⋊ĤHȞL̀Ȟ⋉ | ⋊ĹHȞL̀Ȟ⋉ |
| 3 syllables | ⋊Ĺ L̀ȞH⋉ | ⋊ĤHȞL̀L⋉ | ⋊ĹHȞL̀L⋉ |
| ⋊ĤL̀Ȟ⋉ | ⋊Ĺ L̀L̀L⋉ | ⋊ĤHȞHȞ⋉ | ⋊ĹHȞHȞ⋉ |
| ⋊ĤL̀L⋉ | ⋊Ĺ L̀LH⋉ | ⋊ĤHȞHL⋉ | ⋊ĹHȞHL⋉ |
| ⋊ĤHȞ⋉ | ⋊ĹHȞL̀⋉ | ⋊ĤHL̀L̀Ȟ⋉ | ⋊ĹHL̀L̀Ȟ⋉ |
| ⋊ĤHL⋉ | ⋊ĹHȞH⋉ | ⋊ĤHL̀L̀L⋉ | ⋊ĹHL̀L̀L⋉ |
| ⋊ĹL̀Ȟ⋉ | ⋊Ĺ HL̀⋉ | ⋊ĤHLHȞ⋉ | ⋊ĹHLHȞ⋉ |
| ⋊ĹL̀L⋉ | ⋊ĹLH⋉ | ⋊ĤHLHL⋉ | ⋊ĹHLHL⋉ |
| ⋊ĹHȞ⋉ | | | |
| ⋊ĹHL⋉ | | | |

The grammar, $DG_3$, that produces $DL_3$ is given in (39). $DT_3$ is the same size as T of the seed pattern. As in T, the longest string prohibited by a constraint in $DT_3$ has two symbols, so $DL_3$ also has a *k*-value of two.

(39) $DG_3 = \langle D\Sigma_3, B, DT_3 \rangle$
$D\Sigma_3 = \{Ĥ, Ĺ, Ȟ, L, H, L̀\}$
$B = \{⋊, ⋉\}$
$DT_3 = \{*ĤĤ, *ĤĹ, *ĤȞ, *ĤL, *ĹĤ, *ĹĹ, *ĹȞ, *ĹL, *ȞĤ, *ȞĹ, *ȞȞ, *ȞL, *LĤ, *LĹ, *LȞ, *LL, *ȞĤ, *ȞĹ, *ȞL, *HĤ, *HĹ, *HL̀, *HH, *⋊Ȟ, *⋊L, *⋊L̀, *⋊H\}$

As (40) indicates, the derangement that produces $DL_3$ results in Ĥ and Ĺ being assigned to initial position, Ȟ and L being assigned to non-initial odd-numbered positions, and H and L̀ being assigned to even-numbered positions. In effect, then, the derangement leaves Ĥ, Ĺ, Ȟ, and

H in place but exchanges L̀ and L with respect to their positions in the seed pattern. Since the derangement results in a pair consisting of a light syllable and a heavy syllable with the same degree of stress assigned to initial position, the result is quantity insensitivity in initial position. Since the derangement results in a pair consisting of a light syllable and a heavy syllable with the different degrees of stress assigned to non-initial odd-numbered positions, the result is quantity sensitivity in non-initial odd-numbered positions. Finally, since a pair consisting of a light syllable and a heavy syllable with the different degrees of stress is assigned to even-numbered positions, the result is quantity sensitivity in even-numbered positions.

(40) Distribution of H́, Ĺ, H̀, L̀, H, L in $DL_3$
   a. H́, Ĺ occur in initial position.
   b. H̀, L occur in non-initial odd-numbered positions from the left.
   c. H, L̀ occur in even-numbered positions from the left.

Next are patterns that are partially quantity restricted and partially quantity insensitive. Of the 90 distinct patterns resulting from the derangement of the Maranungku seed pattern, 18 have two quantity-restricted distributions and one quantity-insensitive distribution. Patterns with quantity-restricted distributions of the type discussed here are unattested and implausible.

One of the patterns with two quantity-restricted distributions and one quantity-insensitive distribution results from the derangement that produces $DL_4$. The derangement that produces $DL_4$ is a partial derangement with four fixed points: only the primary stressed heavy syllable symbol and the secondary stressed light syllable symbol have been swapped from their positions in the seed pattern. Under this partial derangement, the initial position must contain a light syllable, making initial position quantity restricted, but the light syllable may optionally carry either a primary stress or a secondary stress. Non-initial odd-numbered syllables must be heavy, making non-initial odd-numbered positions quantity restricted, as well, but these heavy syllables can optionally carry either a primary stress or a secondary stress. Even-numbered syllables are always unstressed and can be either heavy or light, making even-numbered positions quantity insensitive.

(41) $DL_4$ example forms

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊Ĺ⋉ | ⋊ĹHḦH⋉ | ⋊ĹHḦHḦ⋉ | ⋊ỲHḦHḦ⋉ |
| ⋊Ỳ⋉ | ⋊ĹHḦL⋉ | ⋊ĹHḦHỲ⋉ | ⋊ỲHḦHỲ⋉ |
| | ⋊ĹHỲH⋉ | ⋊ĹHḦLḦ⋉ | ⋊ỲHḦLḦ⋉ |
| *2 syllables* | ⋊ĹHỲL⋉ | ⋊ĹHḦLỲ⋉ | ⋊ỲHḦLỲ⋉ |
| ⋊ĹH⋉ | ⋊ĹĹḦH⋉ | ⋊ĹHỲHḦ⋉ | ⋊ỲHỲHḦ⋉ |
| ⋊ĹL⋉ | ⋊ĹĹḦL⋉ | ⋊ĹHỲHỲ⋉ | ⋊ỲHỲHỲ⋉ |
| ⋊ỲH⋉ | ⋊ĹĹỲH⋉ | ⋊ĹHỲLḦ⋉ | ⋊ỲHỲLḦ⋉ |
| ⋊ỲL⋉ | ⋊ĹĹỲL⋉ | ⋊ĹHỲLỲ⋉ | ⋊ỲHỲLỲ⋉ |
| | ⋊ỲHḦH⋉ | ⋊ĹĹḦHḦ⋉ | ⋊ỲĹḦHḦ⋉ |
| 3 syllables | ⋊ỲHḦL⋉ | ⋊ĹĹḦHỲ⋉ | ⋊ỲĹḦHỲ⋉ |
| ⋊ĹHḦ⋉ | ⋊ỲHỲH⋉ | ⋊ĹĹḦLḦ⋉ | ⋊ỲĹḦLḦ⋉ |
| ⋊ĹHỲ⋉ | ⋊ỲHỲL⋉ | ⋊ĹĹḦLỲ⋉ | ⋊ỲĹḦLỲ⋉ |
| ⋊ĹĹḦ⋉ | ⋊ỲĹḦH⋉ | ⋊ĹĹỲHḦ⋉ | ⋊ỲĹỲHḦ⋉ |
| ⋊ĹĹỲ⋉ | ⋊ỲĹḦL⋉ | ⋊ĹĹỲHỲ⋉ | ⋊ỲĹỲHỲ⋉ |
| ⋊ỲHḦ⋉ | ⋊ỲĹỲH⋉ | ⋊ĹĹỲLḦ⋉ | ⋊ỲĹỲLḦ⋉ |
| ⋊ỲHỲ⋉ | ⋊ỲĹỲL⋉ | ⋊ĹĹỲLỲ⋉ | ⋊ỲĹỲLỲ⋉ |
| ⋊ỲĹḦ⋉ | | | |
| ⋊ỲĹỲ⋉ | | | |

The grammar, $DG_4$, that produces $DL_4$ is given in (42). $DT_4$ is the same size as T of the Maranungku pattern. As in T, the longest string prohibited by a constraint in $DT_4$ has two symbols, so $DL_4$ has a *k*-value of two.

(42) $DG_4 = \langle D\Sigma_4, B, DT_4 \rangle$
$D\Sigma_4 = \{$Ỳ, Ĺ, Ỳ, Ḧ, H, L$\}$
$B = \{⋊, ⋉\}$
$DT_4 = \{$*ĹĹ, *ĹỲ, *ĹḦ, *ĹỲ, *ỲĹ, *ỲỲ, *ỲḦ, *ỲỲ, *ḦĹ, *ḦỲ, *ḦḦ, *ḦỲ, *ỲĹ, *ỲỲ, *ỲḦ, *ỲỲ, *ḦĹ, *ḦỲ, *HH, *HL, *LĹ, *LỲ, *LH, *LL, *⋊Ḧ, *⋊Ỳ, *⋊H, *⋊L$\}$

As indicated in (43), the derangement that produces $DL_4$ results in Ỳ and Ĺ being assigned to initial position, Ỳ and Ḧ to non-initial odd-numbered positions, and H and L to even-numbered positions. In effect, then, the derangement leaves Ĺ, Ỳ, H, and L in place with respect to the seed pattern, but it exchanges Ḧ and Ỳ. Since the derangement results in a pair of light syllables with different degrees of stress assigned to initial position, initial position is quantity restricted. Since the derangement results in two heavy syllables with the different degrees of stress assigned to non-initial odd-numbered positions, non-initial odd-numbered positions are quantity restricted, as well. Finally, since an unstressed heavy syllable and an unstressed light syllable with the same degree of stress is assigned to even-numbered positions, even-numbered positions are quantity insensitive.

(43) Distribution of Ḧ, Ĺ, Ỳ, Ỳ, H, L in the $DL_4$ pattern
a. Ỳ, Ĺ occur in initial position.
b. Ỳ, Ḧ occur in non-initial odd-numbered positions from the left.
c. H, L occur in even-numbered positions from the left.

Finally, we consider patterns that are partially quantity restricted and partially quantity sensitive. Of the 90 distinct patterns resulting from the derangement of the Maranungku pattern, 36 have two quantity-restricted distributions and one quantity-sensitive distribution. All 36 patterns are unattested. Given the nature of the quantity restrictedness and quantity sensitivity discussed here, the patterns are all implausible.

One of the patterns with two quantity-restricted distributions and one quantity-sensitive distribution is the result of the derangement that produces $DL_5$ (and seven other derangements). $DL_5$ results from a partial derangement with three fixed points. The primary stressed heavy syllable symbol shifts to replace the secondary stressed light syllable symbol; the secondary stressed light syllable symbol shifts to replace the unstressed heavy syllable symbol; and the unstressed heavy syllable symbol shifts to replace the primary stressed heavy syllable symbol. In the resulting pattern, illustrated in (44), initial syllables are stressless if heavy, but they carry primary stress if light. Non-initial odd-numbered syllables are always heavy, but they may optionally occur with either primary stress or secondary stress. Even-numbered syllables are always light, but they may optionally occur either with secondary stress or without stress.

(44) $DL_5$ example forms

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊Ĺ⋉ | ⋊ĹL̀Ĥ̀L⋉ | ⋊ĹL̀Ĥ̀LĤ⋉ | ⋊HL̀Ĥ̀L̀Ĥ⋉ |
| ⋊H⋉ | ⋊ĹL̀Ĥ̀L⋉ | ⋊ĹL̀Ĥ̀LH̀⋉ | ⋊HL̀Ĥ̀LH̀⋉ |
| | ⋊ĹL̀H̀L⋉ | ⋊ĹL̀ĤLĤ⋉ | ⋊HL̀ĤLĤ⋉ |
| *2 syllables* | ⋊ĹL̀H̀L⋉ | ⋊ĹL̀ĤLH̀⋉ | ⋊HL̀ĤLH̀⋉ |
| ⋊ĹL̀⋉ | ⋊ĹLĤ̀L⋉ | ⋊ĹL̀H̀L̀Ĥ⋉ | ⋊HL̀H̀L̀Ĥ⋉ |
| ⋊ĹL⋉ | ⋊ĹLĤL⋉ | ⋊ĹL̀H̀LH̀⋉ | ⋊HL̀H̀LH̀⋉ |
| ⋊HL̀⋉ | ⋊ĹLH̀L̀⋉ | ⋊ĹL̀H̀LĤ⋉ | ⋊HL̀H̀LĤ⋉ |
| ⋊HL⋉ | ⋊ĹLH̀L⋉ | ⋊ĹL̀H̀LH̀⋉ | ⋊HL̀H̀LH̀⋉ |
| | ⋊HL̀ĤL̀⋉ | ⋊ĹLĤL̀Ĥ⋉ | ⋊HLĤL̀Ĥ⋉ |
| 3 syllables | ⋊HL̀ĤL⋉ | ⋊ĹLĤL̀H̀⋉ | ⋊HLĤL̀H̀⋉ |
| ⋊ĹL̀Ĥ⋉ | ⋊HL̀H̀L̀⋉ | ⋊ĹLĤLĤ⋉ | ⋊HLĤLĤ⋉ |
| ⋊ĹL̀H̀⋉ | ⋊HL̀H̀L⋉ | ⋊ĹLĤLH̀⋉ | ⋊HLĤLH̀⋉ |
| ⋊ĹLĤ⋉ | ⋊HLĤL̀⋉ | ⋊ĹLH̀L̀Ĥ⋉ | ⋊HLH̀L̀Ĥ⋉ |
| ⋊ĹLH̀⋉ | ⋊HLĤL⋉ | ⋊ĹLH̀L̀H̀⋉ | ⋊HLH̀L̀H̀⋉ |
| ⋊HL̀Ĥ⋉ | ⋊HLH̀L̀⋉ | ⋊ĹLH̀LĤ⋉ | ⋊HLH̀LĤ⋉ |
| ⋊HL̀H̀⋉ | ⋊HLH̀L⋉ | ⋊ĹLH̀LH̀⋉ | ⋊HLH̀LH̀⋉ |
| ⋊HLĤ⋉ | | | |
| ⋊HLH̀⋉ | | | |

The grammar in (45), $DG_5$, produces $DL_5$. $DT_5$ is the same size as T of the seed pattern. The longest string prohibited by a constraint in $DT_5$ has two symbols, giving $DL_5$ a *k*-value of two.

(45) $DG_5 = \langle D\Sigma_5, B, DT_5 \rangle$
$D\Sigma_5 = \{H, Ĺ, H̀, Ĥ, L̀, L\}$
$B = \{⋊, ⋉\}$
$DT_5 = \{*ĹĹ, *ĹH, *ĹĤ, *ĹH̀, *HĹ, *HH, *HĤ, *HH̀, *ĤĹ, *ĤH, *ĤĤ, *ĤH̀, *H̀Ĺ, *H̀H, *H̀Ĥ, *H̀H̀, *L̀Ĺ, *L̀H, *L̀L̀, *L̀L, *LĹ, *LH, *LL̀, *LL, *⋊Ĥ, *⋊H̀, *⋊L̀, *⋊L, \}$

As (46) indicates, the derangement that produces $DL_5$ results in H and Ĺ being assigned to initial position, H̀ and Ĥ to non-initial odd-numbered positions, and L̀ and L to even-numbered positions. In effect, then, the derangement leaves Ĺ, H̀, and L in place with respect to the seed pattern and exchanges Ĥ, L̀, and H. Since the derangement results in a heavy syllable and a light syllable with different degrees of stress assigned to initial position, initial position is quantity sensitive. Since the derangement results in a pair of heavy syllables with different degrees of stress assigned to non-initial odd-numbered positions, non-initial odd-numbered positions are quantity restricted. Finally, since a pair of light syllables with different degrees of stress is assigned to even-numbered positions, even-numbered positions are also quantity restricted.

(46) Distribution of Ĥ, Ĺ, H̀, L̀, H, L in the $DL_5$ pattern
   a. H, Ĺ occur in initial position.
   b. H̀, Ĥ occur in non-initial odd-numbered positions from the left.
   c. L̀, L occur in even-numbered positions from the left.

In this section, we have seen how the Maranungku stress pattern is represented with a Σ with six symbols. The six-symbol Σ gives grammars sufficient resources to make minimal, generic syllable weight distinctions. Using Maranungku as the seed pattern, we examined the results of derangement with this expanded alphabet. We saw that 720 partial derangements are possible yielding 90 distinct stress patterns. Except for the deranged pattern that was identical to the original seed pattern, the deranged patterns are unattested, and the vast majority are implausible.

As in the previous section, a greater number of fixed points in a partial derangement does not appear to correspond to a pattern being more plausible. The key factor determining how many and what kinds of problematic phenomena emerge depends on the content of the symbols—their substance—and how the symbols are combined in the distributional assignments.

Focusing on Maranungku as the seed pattern at this point in the discussion offered the opportunity to see how different characteristics—quantity insensitivity, quantity sensitivity, quantity restrictedness—could emerge. Each of the Maranungku distributions is unique in the sense that none overlap with any other distribution. This allowed us to see the different characteristics emerge by assigning different combinations of symbols to particular distributions, making the conditions under which the different characteristics emerge especially transparent. In the next section, we consider Wergaia as the seed pattern, allowing us to examine a case where the characteristics emerge under more subtle circumstances.

### *4. Heavy and light syllables: Wergaia*

In this section, we discuss the results of derangement of Σ = {Ĥ, Ĺ, H̀, L̀, H, L} using the Wergaia pattern rather than the Maranungku pattern as the seed pattern. As we shall see, using Wergaia as the seed pattern increases to 720 the number of distinct patterns that result from derangement. Given the volume of distinct patterns, we will only provide a summary discussion of the results in very general terms. At this point, however, simply registering the number of distinct patterns becomes more important than discussing any of them in detail. There are only 109 patterns in Heinz's (2007, 2009) database, meaning that, at most, 109 of the 720 patterns are attested. Keep in mind that this is the result from a single seed pattern. If we combine these patterns with the 90 resulting from Maranungku discussed in the previous section, then, at most,

109 of 810 patterns are unattested. This is the result from just two seed patterns using a fairly small, coarse-grained syllable-based alphabet.

As (47) illustrates, Wergaia is quite similar to Maranungku. Two main differences are relevant at this stage. The first is that Wergaia has a bimoraic minimal word: monosyllabic words must be heavy syllables. The second is that Wergaia has quantity sensitivity in final syllables: light final syllables cannot be stressed. In Wergaia, then, stress falls on odd-numbered syllables from the left, regardless of weight, except for final syllables. If an odd-numbered final syllable is heavy, it is stressed. If it is light, it is unstressed. The initial stress is the primary stress.

(47) Wergaia pattern

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊H́⋉ | ⋊H́HH̀H⋉ | ⋊H́HH̀HH̀⋉ | ⋊Ĺ HH̀HH̀⋉ |
| | ⋊H́HH̀L⋉ | ⋊H́HH̀HL⋉ | ⋊Ĺ HH̀HL⋉ |
| *2 syllables* | ⋊H́HĹH⋉ | ⋊H́HH̀LH̀⋉ | ⋊Ĺ HH̀LH̀⋉ |
| ⋊H́H⋉ | ⋊H́HĹL⋉ | ⋊H́HH̀LL⋉ | ⋊Ĺ HH̀LL⋉ |
| ⋊H́L⋉ | ⋊H́LH̀H⋉ | ⋊H́HĹHH̀⋉ | ⋊Ĺ HĹHH̀⋉ |
| ⋊Ĺ H⋉ | ⋊H́LH̀L⋉ | ⋊H́HĹHL⋉ | ⋊Ĺ HĹHL⋉ |
| ⋊Ĺ L⋉ | ⋊H́LĹH⋉ | ⋊H́HĹLH̀⋉ | ⋊Ĺ HĹLH̀⋉ |
| | ⋊H́LĹL⋉ | ⋊H́HĹLL⋉ | ⋊Ĺ HĹLL⋉ |
| 3 syllables | ⋊Ĺ HH̀H⋉ | ⋊H́LH̀HH̀⋉ | ⋊Ĺ LH̀HH̀⋉ |
| ⋊H́HH̀⋉ | ⋊Ĺ HH̀L⋉ | ⋊H́LH̀HL⋉ | ⋊Ĺ LH̀HL⋉ |
| ⋊H́HL⋉ | ⋊Ĺ HĹH⋉ | ⋊H́LH̀LH̀⋉ | ⋊Ĺ LH̀LH̀⋉ |
| ⋊H́LH̀⋉ | ⋊Ĺ HĹL⋉ | ⋊H́LH̀LL⋉ | ⋊Ĺ LH̀LL⋉ |
| ⋊H́LL⋉ | ⋊Ĺ LH̀H⋉ | ⋊H́LĹHH̀⋉ | ⋊Ĺ LĹHH̀⋉ |
| ⋊Ĺ HH̀⋉ | ⋊Ĺ LH̀L⋉ | ⋊H́LĹHL⋉ | ⋊Ĺ LĹHL⋉ |
| ⋊Ĺ HL⋉ | ⋊Ĺ LĹH⋉ | ⋊H́LĹLH̀⋉ | ⋊Ĺ LĹLH̀⋉ |
| ⋊Ĺ LH̀⋉ | ⋊Ĺ LĹL⋉ | ⋊H́LĹLL⋉ | ⋊Ĺ LĹLL⋉ |
| ⋊Ĺ LL⋉ | | | |

The grammar responsible for the Wergaia pattern is provided in (48). Note that the longest string prohibited by a constraint in this case has three characters, so the Wergaia pattern has a *k*-value of three.

(48) G = ⟨Σ, B, T⟩
Σ = {H́, Ĺ, H̀, L̀, H, L}
B = {⋊, ⋉}
T = {*H́H́, *H́Ĺ, *H́H̀, *H́L̀, *ĹH́, *ĹĹ, *ĹH̀, *ĹL̀, *H̀H́, *H̀Ĺ, *H̀H̀, *H̀L̀, *L̀H́, *L̀Ĺ, *L̀H̀, *L̀L̀, *HH́, *HĹ, *HH, *LH́, *LĹ, *LH, *⋊H̀, *⋊L̀, *⋊H, *⋊L, *Ĺ⋉, *L̀⋉, *HLH̀, *HLL̀, *HLL, *LLH̀, *LLL̀, *LLL}

The distributions for the Wergaia pattern are given in (49). While Wergaia is similar to Maranungku, its bimoraic minimal word and final quantity sensitivity require a significant revision of the relevant distributions and symbol assignments. Notice that no two symbols are assigned to the same distribution.

(49) Distribution of Ḣ, Ĺ, H̀, L̀, H, L in the Wergaia pattern

a. Ḣ occurs in initial position
b. Ĺ occurs in non-final initial position.
c. H̀ occurs in non-initial odd-numbered positions from the left.
d. L̀ occurs in non-final, non-initial odd-numbered positions from the left.
e. H occurs in even-numbered positions from the left.
f. L occurs in even-numbered positions and final odd-numbered positions from the left.

Despite each symbol being assigned to a distinct distribution, features like quantity insensitivity, quantity sensitivity, and quantity restrictedness can still emerge. Their emergence, however, and the particular characteristics that they emerge with, depend on areas of overlap and areas of non-overlap between the different distributions.

Only one position is mentioned in a single distribution and excluded from all others in (49). Initial position in monosyllabic words is included in the distribution to which Ḣ is assigned, (49a), and is not included in any of the remaining distributions. This results in a type of quantity restrictedness that is not problematic in the way that we saw in the previous section in that it is not accompanied by optionality of stress levels. Whether or not quantity restrictedness without stress level optionality is problematic depends on whether or not the combination of position, syllable weight, and stress level is actually attested. In this case, of course, it is attested. In Wergaia, monosyllabic forms can only be heavy syllables with primary stress. If monosyllabic forms were restricted to a light syllable, or to a heavy syllable with any other stress level, the restriction would be unattested.

In non-minimal forms, every position is mentioned in two distributions. First, initial position in multi-syllabic forms is mentioned in the distribution to which Ḣ is assigned, (49a), and the distribution to which Ĺ is assigned, (49b). Ḣ is assigned to initial positions generally, and Ĺ is assigned to initial positions that are not also final positions. Ḣ can occur in monosyllabic and multisyllabic words then, but Ĺ can occur only in multi-syllabic words. Because two different weights, each with the same level of stress, can occur in initial position in multi-syllabic words, the initial position of multi-syllabic words is quantity insensitive.

Next, quantity insensitivity also arises in two additional sets of positions: medial odd-numbered positions and non-initial even-numbered positions. Medial odd-numbered positions are included both in the distribution to which H̀ is assigned, (49c), and in the distribution to which L̀ is assigned, (49d). Even-numbered syllables are included both in the distribution to which H is assigned, (49e), and the distribution to which L is assigned, (49f). Because two different weights, each with the same level of stress, can occur in these two sets of positions, both sets of positions are quantity insensitive.

Finally, the last location that is mentioned in two distributions is quantity sensitive. The distribution to which H̀ is assigned, (49c), and the distribution to which L is assigned, (49f), both include the final odd-numbered position in non-minimal forms. In this case, two different weights with different degrees of stress can occupy the same location. This makes the final odd-numbered position in non-minimal forms quantity sensitive. This, of course, is an attested location for this type of quantity sensitivity. It is unproblematic.

We turn now to the patterns that result from derangement using Wergaia as the seed pattern. Given the current alphabet, Σ = {Ḣ, Ĺ, H̀, L̀, H, L}, and the distributions for the Wergia

stress pattern given in (49), the number of distinct patterns resulting from derangement is calculated as follows

$$\frac{6!}{1! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} = 720$$

720 is both many times the number of patterns that occur in Heinz's database and far too many to illustrate individually. We will merely mention some problematic phenomena that arise with certain types of distribution assignments.

The type of problematic quantity restrictedness that we saw in the previous section—the type with optionality of stress levels—emerges in deranged languages where two symbols with the same weight but different stress levels are assigned to intersecting distributions. For example, the partial assignment in (50) would lead to initial syllables in multi-syllabic forms being restricted to light syllables, and the light syllables could optionally occur either with secondary stress or without stress.

(50) Distribution of H́, Ĺ, H̀, L̀, H, L in …
   a. L occurs in initial position
   b. L̀ occurs in non-final initial position.
   c. ... occurs in non-initial odd-numbered positions from the left.
   d. ... occurs in non-final, non-initial odd-numbered positions from the left.
   e. ... occurs in even-numbered positions from the left.
   f. ... occurs in even-numbered positions and final odd-numbered positions from the left.

Note that this same distribution assignment also restricts monosyllabic forms to light unstressed syllables. While this type of quantity restrictedness would not involve optionality of stress levels, languages that restrict monosyllabic forms to light unstressed syllables are both unattested and implausible.

Problematic quantity sensitivity emerges in deranged languages where two symbols with different weight and different stress levels are assigned to positions in the intersection of distributions, other than final position. For example, the partial distribution assignment in (51) would lead to medial odd-numbered syllables having secondary stress if heavy and being unstressed if light. The problem here is not the degrees of stress associated with the two weights but the restriction of the effect to this particular distribution.

(51) Distribution of H́, Ĺ, H̀, L̀, H, L in ….
   a. ... occurs in initial position
   b. ... occurs in non-final initial position.
   c. H̀ occurs in non-initial odd-numbered positions from the left.
   d. L occurs in non-final, non-initial odd-numbered positions from the left.
   e. ... occurs in even-numbered positions from the left.
   f. ... occurs in even-numbered positions and final odd-numbered positions from the left.

Problematic quantity sensitivity emerges even in final syllables in non-minimal forms—where isolated quantity-sensitivity is attested—if the light syllable has a higher stress level than the heavy syllable. This is the case given the partial distribution in (52).

(52) Distribution of H́, Ĺ, H̀, L̀, H, L in …
  a. ... occurs in initial position
  b. ... occurs in non-final initial position.
  c. H occurs in non-initial odd-numbered positions from the left.
  d. ... occurs in non-final, non-initial odd-numbered positions from the left.
  e. ... occurs in even-numbered positions from the left.
  f. Ĺ occurs in even-numbered positions and final odd-numbered positions from the left.

With four points of overlap in the distributions and only three symbol combinations that produce quantity-insensitivity, no derangement of the Wergaia seed pattern can result in a purely quantity-insensitive pattern. Every derangement must result in at least one point of distributional intersection that is either quantity sensitive or quantity restricted. Only the derangement that is identical to the seed pattern is attested—meaning that 719 of the deranged patterns are unattested—and the vast majority of these unattested patterns are implausible. In this example, as in the previous examples considered, the degree of scrambling of Σ does not appear to correlate with the plausibility of a deranged pattern. Plausibility depends on the content of the symbols assigned to particular distributions and on the combined content of the symbols in overlapping distributions.

*5. Long vowels and Diphthongs: Maranungku*

In this section and the following, we expand the alphabet again and dig a little bit deeper into the Maranungku and Wergaia patterns. The expanded alphabet will allow us to distinguish between three different types of syllable that might count as heavy: syllables that can count as heavy by virtue of being closed, which we indicate with C, syllables that can count as heavy by virtue of having diphthongs, which we indicate with D, and syllables that can count as heavy by virtue of having long vowels, which we indicate with V. We will continue to indicate syllables that are always light (i.e. open syllables with single short vowels) with L. There will be a symbol combining each syllable type with one of three different degrees of stress. V́, V̀, and V indicate syllables with long vowels that have primary stress, secondary stress, and no stress respectively. D́, D̀, and D indicate syllables with diphthongs that have primary stress, secondary stress, and no stress respectively. Ć, C̀, and C indicate closed syllables that have primary stress, secondary stress, and no stress respectively. Finally, Ĺ, L̀, and L indicate light syllables that have primary stress, secondary stress, and no stress respectively.

While open syllables with short vowels are always light in stress systems that are sensitive to syllable weight, syllables that are closed, syllables that have diphthongs, and syllables that have long vowels are all at least potentially heavy. The ability to distinguish between the different potentially heavy types plays a role in the stress patterns of a number of languages (de Lacy 1997, Gordon 2006, Ryan 2019). This might be because a language has more than two levels of syllable weight, making one type of heavy syllable heavier than another, or it might be because a language treats one or more of the potentially heavy syllable types as light.

The distinctions that arise between the potentially heavy syllable types are asymmetric. In languages that distinguish between syllables with long vowels and syllables with diphthongs, syllables with long vowels are heavier. In languages that distinguish between syllables with long vowels and closed syllables with single short vowels, syllables with long vowels are heavier. In languages that distinguish between syllables with diphthongs and closed syllables with single short vowels, syllables with diphthongs are heavier. As a general rule, languages where stress is sensitive to syllable weight place a greater degree of stress on heavier syllables. A language might require syllables to have a certain minimum weight to qualify for a primary stress, for example, or to qualify for a secondary stress. Another language might require that syllables with a certain weight maintain a minimum stress level—usually secondary stress but sometimes primary stress. Any alphabet that we might use to model stress systems generally must have a way to distinguish between these different options.

Of the three types of potentially heavy syllable included in the twelve-symbol alphabet, Maranungku has only closed syllables. It does not have long vowels or diphthongs. In the example forms in (53), then, the Ḿ, Ḥ, and H symbols from the example Maranungku forms in Section 3 are simply replaced by Ć, Ċ, and C symbols. This yields essentially the same pattern for Maranungku as the pattern illustrated in previous sections. Open and closed syllables can both occur in any position. Initial syllables have primary stress, regardless of whether they are open or closed; non-initial odd-numbered syllables have secondary stress, regardless of whether they are open or closed; and even-numbered syllables are unstressed, regardless of whether they are open or closed.

(53) Maranungku pattern

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊Ć⋉ | ⋊ĆCĊC⋉ | ⋊ĆCĊCĊ⋉ | ⋊ĹCĊCĊ⋉ |
| ⋊Ĺ⋉ | ⋊ĆCĊL⋉ | ⋊ĆCĊCL̀⋉ | ⋊ĹCĊCL̀⋉ |
| | ⋊ĆCL̀C⋉ | ⋊ĆCĊL̀Ċ⋉ | ⋊ĹCĊL̀Ċ⋉ |
| *2 syllables* | ⋊ĆCL̀L⋉ | ⋊ĆCĊL̀L̀⋉ | ⋊ĹCĊL̀L̀⋉ |
| ⋊ĆC⋉ | ⋊ĆLĊC⋉ | ⋊ĆCL̀CĊ⋉ | ⋊ĹCL̀CĊ⋉ |
| ⋊ĆL⋉ | ⋊ĆLĊL⋉ | ⋊ĆCL̀CL̀⋉ | ⋊ĹCL̀CL̀⋉ |
| ⋊ĹC⋉ | ⋊ĆLL̀C⋉ | ⋊ĆCL̀LĊ⋉ | ⋊ĹCL̀LĊ⋉ |
| ⋊ĹL⋉ | ⋊ĆLL̀L⋉ | ⋊ĆCL̀L̀L⋉ | ⋊ĹCL̀L̀L⋉ |
| | ⋊ĹCĊC⋉ | ⋊ĆLĊCĊ⋉ | ⋊ĹLĊCĊ⋉ |
| 3 syllables | ⋊ĹCĊL⋉ | ⋊ĆLĊCL̀⋉ | ⋊ĹLĊCL̀⋉ |
| ⋊ĆCĊ⋉ | ⋊ĹCL̀C⋉ | ⋊ĆLĊL̀Ċ⋉ | ⋊ĹLĊL̀Ċ⋉ |
| ⋊ĆCL̀⋉ | ⋊ĹCL̀L⋉ | ⋊ĆLĊL̀L̀⋉ | ⋊ĹLĊL̀L̀⋉ |
| ⋊ĆLĊ⋉ | ⋊ĹLĊC⋉ | ⋊ĆLL̀CĊ⋉ | ⋊ĹLL̀CĊ⋉ |
| ⋊ĆLL̀⋉ | ⋊ĹLĊL⋉ | ⋊ĆLL̀CL̀⋉ | ⋊ĹLL̀CL̀⋉ |
| ⋊ĹCĊ⋉ | ⋊ĹLL̀C⋉ | ⋊ĆLL̀L̀Ċ⋉ | ⋊ĹLL̀L̀Ċ⋉ |
| ⋊ĹCL̀⋉ | ⋊ĹLL̀L⋉ | ⋊ĆLL̀L̀L̀⋉ | ⋊ĹLL̀L̀L̀⋉ |
| ⋊ĹLĊ⋉ | | | |
| ⋊ĹLL̀⋉ | | | |

The grammar responsible for the Maranungku pattern as represented with the twelve-symbol alphabet is given in (54). There are two key features of the grammar to notice. First, the expansion of the alphabet does not affect the pattern's *k*-value. The longest string prohibited by a

constraint has two characters, so the pattern still has a *k*-value of two. Second, the grammar excludes six of the alphabet's twelve characters from appearing in Maranungku forms. Each of the first six constraints listed in T contains just one symbol (one of the symbols not found in Maranungku) and the grammar excludes any form that contains that symbol.

(54) G = ⟨Σ, B, T⟩
Σ = {V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L}
B = {⋊, ⋉}
T = {*V́, *D́, *V̀, *D̀, *V, *D, *ĆĆ, *ĆĹ, *ĆC̀, *ĆL̀, *ĹĆ, *ĹĹ, *ĹC̀, *ĹL̀, *C̀Ć, *C̀Ĺ, *C̀C̀, *C̀L̀, *L̀Ć, *L̀Ĺ, *L̀C̀, *L̀L̀, *CĆ, *CĹ, *CC, *CL, *LĆ, *LĹ, *LC, *LL, *⋊C̀, *⋊L̀, *⋊C, *⋊L}

With the twelve-symbol alphabet, the distributions of the Maranungku stress pattern expand to include an additional distribution for the excluded symbols. The excluded symbols are assigned to a null distribution, as in (55d). Distributions (55a-c) are the same distributions that we have seen in previous representations of Maranungku. Ĺ, L̀, and L are assigned to the same distributions that they were assigned to in Section 3, and Ć, C̀, and C replace Ĥ, Ĥ, and H, respectively, in the distributions from Section 3.

(55) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L in Maranungku
a. Ć, Ĺ occur in initial position.
b. C̀, L̀ occur in non-initial odd-numbered positions from the left.
c. C, L occur in even-numbered positions from the left.
d. V́, D́, V̀, D̀, V, D occur in no position.

Given the current alphabet, Σ = {V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L}, and the distributions for the Maranungku stress pattern given in (55), the number of distinct new patterns resulting from derangement is calculated as follows

$$\frac{12!}{2! \cdot 2! \cdot 2! \cdot 6!} = 83{,}160$$

While the number of deranged patterns is much too large to explore them in detail, it is worth pointing out a few of the problematic predictions that can arise from the derangement of Maranungku with the twelve-symbol alphabet.

Derangements where the non-null distributions have just two syllable types assigned to them, as in (55), yield the same types of patterns that we saw in Section 3. For example, derangements with the C and L syllable types assigned to the non-null distributions yield the same types of quantity-insensitive, quantity-sensitive, quantity-restricted, and combined patterns that we saw in Section 3 (with C replacing H in the patterns). Derangement produces similar patterns when D and L syllable types are assigned to the non-null distributions (with D replacing H in the patterns) and when V and L syllable types are assigned to the non-null distributions (with V replacing H in the patterns). It also produces similar patterns when V and D, V and C, or D and C syllable types are assigned to the non-null distributions, replacing both H and L in the patterns from Section 3, but these last three combinations would each have the additional

problematic quirk of creating languages that did not allow light syllables. This is a problematic prediction that we did not see with previous alphabets.

Another problematic prediction that we have not seen before but that emerges with this particular alphabet and seed pattern is the prediction of languages without stressless syllables. This happens whenever a derangement assigns all four of the unstressed syllable symbols to the null distribution. The distribution assignment for $DL_1$ in (56) is one example. $DL_1$, would be quantity restricted in each of its non-null distributions. Initial position would only have syllables with long vowels, but they might optionally have either primary or secondary stress. Odd-numbered non-initial positions would only have syllables with diphthongs, but they might optionally have either primary or secondary stress. Even-numbered positions would only have closed syllables, but they might optionally appear with either primary stress or secondary stress. Note that this particular distribution assignment also results in a language without light syllables.

(56) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, Ć, L̀, V, D, C, L in $DL_1$
   a. V́, V̀ occur in initial position.
   b. D́, D̀ occur in non-initial odd-numbered positions from the left.
   c. Ć, Ć occur in even-numbered positions from the left.
   d. Ĺ, L̀, C, V, D, L occur in no position.

The distribution assignment for $DL_2$ in (57) is another example of a pattern without stressless syllables. $DL_2$ exhibits a sort of quantity restrictedness in that the non-null distributions are not assigned the same set of syllable types. Each non-null distribution is also quantity sensitive. Initial position allows syllables with long vowels and syllables with diphthongs. The former have primary stress, but the latter have secondary stress. Non-initial odd-numbered positions allow syllables with diphthongs and closed syllables. The former have primary stress, but the latter have secondary stress. Even-numbered positions allow closed syllables and light syllables. The former have primary stress, but the latter have secondary stress.

(57) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, Ć, L̀, V, D, C, L in $DL_2$
   a. V́, D̀ occur in initial position.
   b. D́, Ć occur in non-initial odd-numbered positions from the left.
   c. Ć, L̀ occur in even-numbered positions from the left.
   d. Ĺ, V̀, C, V, D, L occur in no position.

We might go on and on listing the strange predictions that emerge from derangement of the twelve-symbol alphabet and the Maranungku seed pattern, but this brief examination should be more than sufficient. There can be no doubt that the vast majority of the patterns yielded in this case, at least 83,051 of 83,160, are unattested. Next, we consider the case of the Wergaia pattern represented with the twelve-symbol alphabet.

### *6. Long vowels and Diphthongs: Wergaia*

As we saw in Section 4, quantity sensitivity in Wergaia is limited to final position. Its weight distinction is a simple two-way heavy vs. light distinction. Odd-numbered final syllables can be stressed only if they are heavy; if they are light, they are unstressed. Wergaia is not one of the patterns where the distinction between the three different types of potentially heavy syllables

affects the distribution of the different levels of stress. Syllables with long vowels are not more likely to be stressed than closed syllables, for example. Each of the three potential heavy syllable types simply count as heavy. Due to positional restrictions, however, the three different types of heavy syllable do not participate equally in Wergaia's limited quantity sensitivity. While closed syllables might occur in any position, syllables with diphthongs must be either initial or final, and syllables with long vowels must be initial (or equivalently, since primary stress is always initial, must carry primary stress). Since Wergaia's quantity sensitivity is limited to final position, and syllables with long vowels are in final position only in monosyllabic forms, syllables with long vowels only participate in quantity sensitivity in monosyllabic forms. Because syllables with diphthongs and closed syllables can occur in final position in forms of any length, they can participate in quantity sensitivity in forms of any length.

The Wergaia pattern represented with the twelve-symbol alphabet is illustrated in (58). Stress occurs automatically on every odd-numbered syllable except the final syllable. Odd numbered final syllables are stressed only if heavy; if they are light, they are unstressed. The initial stress is the primary stress. Monosyllabic forms must be heavy, but any position in longer forms might contain a heavy syllable or a light syllable. As mentioned above, however, the different types of heavy syllables do not have the same distributions. Closed syllables can occur in any position, but syllables with diphthongs are limited to peripheral position, and syllables with long vowels are limited to initial position.

(58) Wergaia pattern

| *1 syllable* | *4 syllables* | *5 syllables* | |
|---|---|---|---|
| ⋊V́⋉ | ⋊V́CC̀D⋉ | ⋊V́CC̀CD̀⋉ | ⋊ĆCC̀CD̀⋉ |
| ⋊D́⋉ | ⋊V́CC̀C⋉ | ⋊V́CC̀CC̀⋉ | ⋊ĆCC̀CC̀⋉ |
| ⋊Ć⋉ | ⋊V́CC̀L⋉ | ⋊V́CC̀CL⋉ | ⋊ĆCC̀CL⋉ |
| | ⋊V́CL̀D⋉ | ⋊V́CC̀LD̀⋉ | ⋊ĆCC̀LD̀⋉ |
| *2 syllables* | ⋊V́CL̀C⋉ | ⋊V́CC̀LC̀⋉ | ⋊ĆCC̀LC̀⋉ |
| ⋊V́D⋉ | ⋊V́CL̀L⋉ | ⋊V́CC̀LL⋉ | ⋊ĆCC̀LL⋉ |
| ⋊V́C⋉ | ⋊V́LC̀D⋉ | ⋊V́CL̀CD̀⋉ | ⋊ĆCL̀CD̀⋉ |
| ⋊V́L⋉ | ⋊V́LC̀C⋉ | ⋊V́CL̀CC̀⋉ | ⋊ĆCL̀CC̀⋉ |
| ⋊D́D⋉ | ⋊V́LC̀L⋉ | ⋊V́CL̀CL⋉ | ⋊ĆCL̀CL⋉ |
| ⋊D́C⋉ | ⋊V́LL̀D⋉ | ⋊V́CL̀LD̀⋉ | ⋊ĆCL̀LD̀⋉ |
| ⋊D́L⋉ | ⋊V́LL̀C⋉ | ⋊V́CL̀LC̀⋉ | ⋊ĆCL̀LC̀⋉ |
| ⋊ĆD⋉ | ⋊V́LL̀L⋉ | ⋊V́CL̀LL⋉ | ⋊ĆCL̀LL⋉ |
| ⋊ĆC⋉ | ⋊D́CC̀D⋉ | ⋊V́LC̀CD̀⋉ | ⋊ĆLC̀CD̀⋉ |
| ⋊ĆL⋉ | ⋊D́CC̀C⋉ | ⋊V́LC̀CC̀⋉ | ⋊ĆLC̀CC̀⋉ |
| ⋊Ĺ D⋉ | ⋊D́CC̀L⋉ | ⋊V́LC̀CL⋉ | ⋊ĆLC̀CL⋉ |
| ⋊ĹC⋉ | ⋊D́CL̀D⋉ | ⋊V́LC̀LD̀⋉ | ⋊ĆLC̀LD̀⋉ |
| ⋊ĹL⋉ | ⋊D́CL̀C⋉ | ⋊V́LC̀LC̀⋉ | ⋊ĆLC̀LC̀⋉ |
| | ⋊D́CL̀L⋉ | ⋊V́LC̀LL⋉ | ⋊ĆLC̀LL⋉ |
| 3 syllables | ⋊D́LC̀D⋉ | ⋊V́LL̀CD̀⋉ | ⋊ĆLL̀CD̀⋉ |
| ⋊V́CD̀⋉ | ⋊D́LC̀C⋉ | ⋊V́LL̀CC̀⋉ | ⋊ĆLL̀CC̀⋉ |
| ⋊V́CC̀⋉ | ⋊D́LC̀L⋉ | ⋊V́LL̀CL⋉ | ⋊ĆLL̀CL⋉ |
| ⋊V́CL⋉ | ⋊D́LL̀D⋉ | ⋊V́LL̀LD̀⋉ | ⋊ĆLL̀LD̀⋉ |
| ⋊V́LD̀⋉ | ⋊D́LL̀C⋉ | ⋊V́LL̀LC̀⋉ | ⋊ĆLL̀LC̀⋉ |
| ⋊V́LC̀⋉ | ⋊D́LL̀L⋉ | ⋊V́LL̀LL⋉ | ⋊ĆLL̀LL⋉ |

⋊ÁLL⋉ ⋊ÁCÈD⋉ ⋊ÁCÈCÈ⋉ ⋊ÁCÈCÈD⋉
⋊ÁCÈ⋉ ⋊ÁCÈC⋉ ⋊ÁCÈCÈC⋉ ⋊ÁCÈCÈC⋉
⋊ÁCÈC⋉ ⋊ÁCÈL⋉ ⋊ÁCÈCL⋉ ⋊ÁCÈCL⋉
⋊ÁCL⋉ ⋊ÁCÈÈD⋉ ⋊ÁCÈCÈD⋉ ⋊ÁCÈCÈD⋉
⋊ÁLÈ⋉ ⋊ÁCÈÈC⋉ ⋊ÁCÈCÈC⋉ ⋊ÁCÈCÈC⋉
⋊ÁLC⋉ ⋊ÁCÈÈL⋉ ⋊ÁCÈCÈL⋉ ⋊ÁCÈCÈL⋉
⋊ÁLL⋉ ⋊ÁCÈÈÈD⋉ ⋊ÁCÈCÈÈD⋉ ⋊ÁCÈCÈÈD⋉
⋊ÁCÈD⋉ ⋊ÁCÈÈÈC⋉ ⋊ÁCÈCÈÈC⋉ ⋊ÁCÈCÈÈC⋉
⋊ÁCÈC⋉ ⋊ÁCÈÈÈL⋉ ⋊ÁCÈCÈÈL⋉ ⋊ÁCÈCÈÈL⋉
⋊ÁCL⋉ ⋊ÁCÈÈÈÈD⋉ ⋊ÁCÈCÈÈÈD⋉ ⋊ÁCÈCÈÈÈD⋉
⋊ÁCÈÈD⋉ ⋊ÁCÈÈÈÈC⋉ ⋊ÁCÈCÈÈÈC⋉ ⋊ÁCÈCÈÈÈC⋉
⋊ÁCÈC⋉ ⋊ÁCÈÈÈÈL⋉ ⋊ÁCÈCÈÈÈL⋉ ⋊ÁCÈCÈÈÈL⋉
⋊ÁCÈÈL⋉ ⋊ÁÉCÈD⋉ ⋊ÁCÈÉCÈ⋉ ⋊ÁCÈÉCÈD⋉
⋊ÁÉCÈ⋉ ⋊ÁÉCÈC⋉ ⋊ÁCÈÉCÈC⋉ ⋊ÁCÈÉCÈC⋉
⋊ÁÉCÈC⋉ ⋊ÁÉCÈL⋉ ⋊ÁCÈÉCÈL⋉ ⋊ÁCÈÉCÈL⋉
⋊ÁÉCL⋉ ⋊ÁÉCÈÈD⋉ ⋊ÁCÈÉCÈÈD⋉ ⋊ÁCÈÉCÈÈD⋉
⋊ÁÉLÈ⋉ ⋊ÁÉCÈÈC⋉ ⋊ÁCÈÉCÈÈC⋉ ⋊ÁCÈÉCÈÈC⋉
⋊ÁÉLC⋉ ⋊ÁÉCÈÈL⋉ ⋊ÁCÈÉCÈÈL⋉ ⋊ÁCÈÉCÈÈL⋉
⋊ÁÉLL⋉ ⋊ÁÉCÈÈÈD⋉ ⋊ÁCÈÉCÈÈÈD⋉ ⋊ÁCÈÉCÈÈÈD⋉
⋊ÁÉLÈD⋉ ⋊ÁÉCÈÈÈC⋉ ⋊ÁCÈÉCÈÈÈC⋉ ⋊ÁCÈÉCÈÈÈC⋉
⋊ÁÉLÈC⋉ ⋊ÁÉCÈÈÈL⋉ ⋊ÁCÈÉCÈÈÈL⋉ ⋊ÁCÈÉCÈÈÈL⋉
⋊ÁÉLÈÈD⋉ ⋊ÁÉCÈÉCÈ⋉ ⋊ÁCÈÉÉCÈD⋉ ⋊ÁCÈÉÉCÈD⋉
⋊ÁÉLÈÈC⋉ ⋊ÁÉCÈÉCÈC⋉ ⋊ÁCÈÉÉCÈC⋉ ⋊ÁCÈÉÉCÈC⋉
⋊ÁÉLÈÈÈL⋉ ⋊ÁÉCÈÉCÈÈL⋉ ⋊ÁCÈÉÉCÈÈL⋉ ⋊ÁCÈÉÉCÈÈL⋉
⋊ÁÉCÈÈÈD⋉ ⋊ÁCÈÉÉÉCÈD⋉ ⋊ÁCÈÉÉÉCÈD⋉
⋊ÁÉCÈÈÈC⋉ ⋊ÁCÈÉÉÉCÈC⋉ ⋊ÁCÈÉÉÉCÈC⋉
⋊ÁÉCÈÈÈL⋉ ⋊ÁCÈÉÉÉCÈL⋉ ⋊ÁCÈÉÉÉCÈL⋉
⋊ÁÉCÈÈÈÈD⋉ ⋊ÁCÈÉÉÉÉCÈD⋉ ⋊ÁCÈÉÉÉÉCÈD⋉
⋊ÁÉCÈÈÈÈC⋉ ⋊ÁCÈÉÉÉÉCÈC⋉ ⋊ÁCÈÉÉÉÉCÈC⋉
⋊ÁÉCÈÈÈÈL⋉ ⋊ÁCÈÉÉÉÉCÈL⋉ ⋊ÁCÈÉÉÉÉCÈL⋉

The grammar in (59) is responsible for the Wergaia pattern in (58). The longest string prohibited by a constraint has three characters, so the Wergaia pattern still has a *k*-value of three even when represented with the twelve-symbol alphabet. As in Maranungku, some of the available symbols do not occur in Wergaia. In Wergaia, the non-occurring symbols are È and V. Syllables with long vowels that have secondary stress or that are unstressed do not occur in the Wergaia pattern. The grammar in (59) implements this exclusion through the first two constraints listed in T, simply prohibiting any string that contains either È or V.

(59) G = ⟨Σ, B, T⟩
Σ = {Á, É, Ć, Ĺ, È, Ì, Ć, Ĺ, V, D, C, L}
B = {⋊, ⋉}
T = {*È, *V, *ÁV, *ÁÉ, *ÁĆ, *ÁĹ, *ÁÌ, *ÁÈ, *ÁĹ, *ÉÁ, *ÉÉ, *ÉĆ, *ÉĹ, *ÉÌ, *ÉÈ, *ÉĆ, *ÉĹ, *ÉÌ, *ĆÁ, *ĆÉ, *ĆĆ, *ĆĹ, *ĆÌ, *ĆÈ, *ĆĆ, *ĆĹ, *ĹÁ, *ĹÉ, *ĹĆ, *ĹĹ, *ĹÌ, *ĹÈ, *ĹĹ, *ÌÁ, *ÌÉ, *ÌĆ, *ÌĹ, *ÌÌ, *ÌÈ, *ÌĆ, *ÌĹ, *ÌÌ, *ÈÁ, *ÈÉ, *ÈĆ, *ÈĹ, *ÈÌ, *ÈÈ, *ÈĆ, *ÈĹ, *ÈÌ, *ÈÁ, *ÈÉ, *ÈĆ, *ÈĹ, *ÈÌ, *ÈÈ, *ÈĆ, *ÈĹ, *ÈÌ, *ÁÉ, *ÁĆ, *ÁĹ, *ÁÌ, *ÁÈ

than it did in the case of Maranungku. The number of distributions necessary has nearly doubled. The only two distributions with multiple symbols assigned to them are the initial position distribution, (60a), which has all of the heavy syllable types with primary stress assigned to it, and the null distribution, (60i), which has syllables with long vowels that have either secondary stress or no stress assigned to it.

(60) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, Ć, L̀, V, D, C, L in the Wergaia pattern
   a. V́, D́, Ć occur in initial position
   b. Ĺ occurs in non-final initial position.
   c. D̀ occurs in non-initial final odd-numbered positions from the left.
   d. Ć occurs in non-initial odd-numbered positions from the left.
   e. L̀ occurs in non-final non-initial odd-numbered positions from the left.
   f. D occurs in final even-numbered positions from the left.
   g. C occurs in even-numbered positions from the left.
   h. L occurs in even-numbered positions and final odd-numbered positions from the left.
   i. V̀, V occur in no positions.

One position is mentioned in a single distribution and excluded from all others in (60). Initial position in monosyllabic words is included in the distribution to which V́, D́, and Ć are assigned, (60a), and is not included in any of the remaining distributions. This has the effect of restricting monosyllabic forms to heavy syllables with primary stress.

In non-minimal forms, every position is mentioned in at least two distributions. First, initial position in multi-syllabic forms is mentioned in the distribution to which V́, D́, and Ć are assigned, (60a) and the distribution to which Ĺ is assigned, (60b). Because symbols representing two different weights, each with the same level of stress, can occur in initial position in longer forms, initial position in longer forms is quantity insensitive.

Quantity insensitivity arises in three additional sets of positions. First, medial odd-numbered positions are included both in the distribution to which Ć is assigned, (60d), and in the distribution to which L̀ is assigned, (60e). Second, non-final even-numbered syllables are included both in the distribution to which C is assigned, (60g), and the distribution to which L is assigned, (60h). Third, final even-numbered syllables are included in the distribution to which D is assigned, (60f), the distribution to which C is assigned, (60g), and the distribution to which L is assigned, (60h). Because two different weights, each with the same stress level, can occur in these three sets of positions, the positions in all three sets are quantity insensitive.

Finally, the last position that is mentioned in multiple distributions is quantity sensitive. Final odd-numbered syllables of non-minimal forms are included in the distribution to which D̀ is assigned, (60c), the distribution to which Ć is assigned, (60d), and the distribution to which L is assigned, (60h). Since two different weights with different stress levels can occupy the final odd-numbered position in non-minimal forms, the final odd-numbered position of non-minimal forms is quantity sensitive.

We turn now to the patterns that result from derangement. Given the current alphabet, $\Sigma$ = {V́, D́, Ć, Ĺ, V̀, D̀, Ć, L̀, V, D, C, L}, and the distributions for the Wergaia pattern given in (60), the number of distinct patterns resulting from derangement is calculated as follows

$$\frac{12!}{3! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 2!} = 39{,}916{,}800$$

Within the distinct patterns produced by derangement, we find more extreme versions of phenomena seen in earlier sections. Of course, given the very large number of patterns, it is not possible to explore them in any detail. We will only point to a few cases that illustrate issues not seen in the previous sections.

The distribution assignments in (60) are the first time that we have encountered the possibility of three different symbols occurring in the same position, either because three different symbols are assigned to one distribution or because three different distributions overlap. There is even one case where multiple symbol assignment and distribution overlap combine to allow the possibility of four symbols occurring in the same position. These situations allow for new possibilities under derangement.

In the partial distribution assignment in (61), for example, V́, V̀, and V are all assigned to the initial position, and Ĺ is assigned to non-final initial position. Initial position in minimal forms, then, is quantity restricted. Only syllables with long vowels can occur in this position, and they can optionally appear with any of the three degrees of stress. Similar situations in previous sections only allowed a choice between two degrees of stress. Initial position in non-minimal forms would also allow syllables with long vowels with any degree of stress, but it would also allow light syllables with primary stress. We might think of initial position as having a type of quantity sensitivity where stress level optionality is allowed for the heavier syllable type but not for the lighter syllable type.

(61) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L in …
- a. V́, V̀, V occur in initial position
- b. Ĺ occurs in non-final initial position.
- c. … occurs in non-initial final odd-numbered positions from the left.
- d. … occurs in non-initial odd-numbered positions from the left.
- e. … occurs in non-final non-initial odd-numbered positions from the left.
- f. … occurs in final even-numbered positions from the left.
- g. … occurs in even-numbered positions from the left.
- h. .. occurs in even-numbered positions and final odd-numbered positions from the left.
- i. … occur in no positions.

The slightly different partial distribution assignment in (62) creates a very different situation. Ć, D̀, and V are all assigned to initial position, and Ĺ is assigned to non-final initial position. Initial position in minimal forms, then, is quantity sensitive, but it is quantity sensitive in a way that runs contrary to the prominence scale for syllable types discussed above. Closed syllables appear with primary stress; syllables with diphthongs appear with secondary stress; and syllables with long vowels appear without stress. This is exactly opposite of what it expected. In initial position in non-minimal forms, light syllables may also appear. This position is quantity sensitive as well with light syllables patterning with closed syllables at the top of the prominence hierarchy.

(62) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, Ć, L̀, V, D, C, L in ...
   a. Ć, D̀, V occur in initial position
   b. Ĺ occurs in non-final initial position.
   c. … occurs in non-initial final odd-numbered positions from the left.
   d. … occurs in non-initial odd-numbered positions from the left.
   e. … occurs in non-final non-initial odd-numbered positions from the left.
   f. … occurs in final even-numbered positions from the left.
   g. … occurs in even-numbered positions from the left.
   h. … occurs in even-numbered positions and final odd-numbered positions from the left.
   i. … occur in no positions.

The partial distribution assignment in (63) illustrates a case where overlapping distributions create a set of positions where two syllables can appear and a position where three symbols can appear. Ć appears in final even-numbered positions; C̀ appears in even-numbered positions; and C appears in even-numbered positions and final odd-numbered positions. The result is that non-final even-numbered positions are quantity restricted. Only closed syllables can occur in these positions, but they may optionally appear with secondary stress or without stress. The final even-numbered position is also quantity restricted. Only closed syllables may occur in this position, but they may optionally appear with any degree of stress.

(63) Distribution of V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L in ...
   a. … occur in initial position
   b. … occurs in non-final initial position.
   c. … occurs in non-initial final odd-numbered positions from the left.
   d. … occurs in non-initial odd-numbered positions from the left.
   e. … occurs in non-final non-initial odd-numbered positions from the left.
   f. Ć occurs in final even-numbered positions from the left.
   g. C̀ occurs in even-numbered positions from the left.
   h. C occurs in even-numbered positions and final odd-numbered positions from the left.
   i. … occur in no positions.

We could continue listing the problematic predictions that emerge from derangement of the twelve-symbol alphabet and the Wergaia seed pattern, but there is little point in doing so. The vast majority of distinct patterns yielded through derangement are unattested. More concretely, of the 39,916,800 phonologically distinct stress patterns resulting from derangement, at least 39,916,691 are unattested. We now turn to the formal proof of the derangement method's validity for Strictly Local grammars.

*7. Proof: Transformed Strictly Local Grammars Yield Transformed Stringsets*

*7.1 Substring Relations are Preserved Under Alphabet Permutation*

*7.1.1 Function Inverses and Bijections*

For any function $f: A \to B$, define the inverse of $f$, $f^{-1}$, to be the relation on $Range(f) \times A$ such that $(\forall a \in A)\big(f^{-1}\big(f(a)\big) = a\big)$ is true.

**Theorem 1**: If $f: A \to B$ is a bijection, then $f^{-1}: B \to A$, is a bijection.
**Proof**
$f$ is an injection, so $f^{-1}$ is an injection from $Range(f)$ to $A$.
$f$ is a surjection, so $Range(f) = B$, and therefore $f^{-1}$ is an injection from $B$ to $A$.
$f$ is a function on $A$, so $f^{-1}$ must be a surjection onto $A$.
$f^{-1}$ is both an injection and a surjection, therefore it is a bijection.
**End Proof**

**Corollary 1**: $f^{-1}\big(f(a)\big) = a$ and $f\big(f^{-1}(b)\big) = b$, where $f: A \to B, a \in A, b \in B$.

*7.1.2 Alphabet Permutations*

Let Σ be a finite alphabet of symbols, and let $\pi: \Sigma \to \Sigma$ be a bijection on Σ. Then $\pi$ is an **alphabet permutation** on Σ. The inverse of alphabet permutation $\pi$ is denoted $\pi^{-1}$. Because $\pi$ is a bijection, $\pi^{-1}$ is also a bijection, and therefore also an alphabet permutation.

*7.1.3 String Substitutions*

It will be convenient at times to positionally index the symbols of a string. Let $w$ be a string over the alphabet Σ. We can describe $w$ as

$$w = w_1 w_2 \ldots w_{|w|}$$

where $w_1$ is the first segment of $w$, $w_2$ is the second segment of $w$, and so forth. $|w|$ denotes the length of $w$, so $w_{|w|}$ is the $w^{\text{th}}$ (and final) segment of $w$.

Let $sub(x, w)$ denote the proposition that $x$ is a substring of $w$. Using the definition of substring, where $w, x, y, z \in \Sigma^*$,

$$sub(x, w) = (\exists y, z)(w = yxz)$$

Given $\pi$ an alphabet permutation on Σ, we will call $\beta: \Sigma^* \to \Sigma^*$ **the string substitution for $\pi$**. With $w \in \Sigma^*$, $\beta$ is defined as

$$\beta(w) = \begin{cases} \lambda & \text{if } w = \lambda \text{ (the empty string)} \\ \pi(w_1)\pi(w_2) \ldots \pi\big(w_{|w|}\big) & \text{if } w \neq \lambda \end{cases}$$

In other words, $\beta(w)$ is the result of applying alphabet permutation $\pi$ to each symbol of $w$. It follows directly that $\beta(w)$ is length-preserving, that is, $|\beta(w)| = |w|$.

**Theorem 2**: Let $\pi$ be an alphabet permutation on alphabet Σ, and $\beta$ be the string substitution for $\pi$. $\beta$ is a bijection.

**Proof**

First, show that $\beta$ is an injection. Suppose that $\beta(v) = \beta(w)$.

If $\beta(v) = \beta(w) = \lambda$, then $v = w = \lambda$. Thus, $v = w$.

If $\beta(v) \neq \lambda$, then

$$\beta(v) = \pi(v_1)\pi(v_2) \dots \pi\left(v_{|v|}\right)$$
$$\beta(w) = \pi(w_1)\pi(w_2) \dots \pi\left(w_{|w|}\right)$$

Because $\beta(v) = \beta(w)$, it follows that $\pi(v_1) = \pi(w_1)$, $\pi(v_2) = \pi(w_2)$, and so forth. $\pi$ is a bijection, so $v_1 = w_1$, $v_2 = w_2$, and so forth. Thus, $v = w$.

$\left(\beta(v) = \beta(w)\right) \rightarrow (v = w)$, therefore $\beta$ is an injection.

Second, show that $\beta$ is a surjection. Let $v \in \Sigma^*$.

If $v = \lambda$, then let $w = \lambda$. By definition, $\beta(\lambda) = \lambda$. Thus, $\beta(w) = v$.

If $v \neq \lambda$, then let $w = \pi^{-1}(v_1)\pi^{-1}(v_2) \dots \pi^{-1}\left(v_{|v|}\right)$.

Then $\beta(w) = \pi\left(\pi^{-1}(v_1)\right)\pi\left(\pi^{-1}(v_2)\right) \dots \pi\left(\pi^{-1}\left(v_{|v|}\right)\right)$. By Corollary 1, $\pi\left(\pi^{-1}(v_i)\right) = v_i$ , so $\beta(w) = v_1 v_2 \dots v_{|v|} = v$.

$(\forall v \exists w)(\beta(w) = v)$. Therefore, $\beta$ is a surjection.

$\beta$ is both an injection and a surjection, therefore $\beta$ is a bijection.

**End Proof**

**Corollary 2**: $\beta^{-1}(w) = \pi^{-1}(w_1)\pi^{-1}(w_2) \dots \pi^{-1}\left(w_{|w|}\right)$.

**Proof**

Let $q(w) = \pi^{-1}(w_1)\pi^{-1}(w_2) \dots \pi^{-1}\left(w_{|w|}\right)$. Then

$$\begin{aligned} q\left(\beta(w)\right) &= q\left(\pi(w_1)\pi(w_2) \dots \pi\left(w_{|w|}\right)\right) \\ &= \pi^{-1}\left(\pi(w_1)\right)\pi^{-1}\left(\pi(w_2)\right) \dots \pi^{-1}\left(\pi\left(w_{|w|}\right)\right) \\ &= w_1 w_2 \dots w_{|w|} \\ &= w \end{aligned}$$

We have that $(\forall w \in \Sigma^*)\left(q\left(\beta(w)\right) = w\right)$. Therefore $q$ is the inverse of $\beta$: $\beta^{-1}(w) = q(w)$.

**End Proof**

**Corollary 3**: $\beta^{-1}$ is the string substitution for the alphabet permutation $\pi^{-1}$.

### *7.1.4 The Proof of Substring Preservation under String Substitution*

Here, we prove that the substring relation is preserved under string substitution, that is, that $x$ is a substring of $w$ if and only if substituted $x$ is a substring of substituted $w$.

**Theorem 3**: Let $x, w \in \Sigma^*$, $\pi$ be a permutation on Σ, and $\beta$ the string substitution for $\pi$. Then

$$sub(x, w) \leftrightarrow sub\left(\beta(x), \beta(w)\right)$$

**Proof**

**The If Case**: $sub(x, w) \rightarrow sub\left(\beta(x), \beta(w)\right)$

$sub(x, w)$ entails that $w = yxz$ for some $y, z \in \Sigma^*$. Thus,

$$w = y_1 y_2 \dots y_{|y|} x_1 x_2 \dots x_{|x|} z_1 z_2 \dots z_{|z|}$$

By the definition of $\beta$,

$$\beta(x) = \underline{\pi(x_1)\pi(x_2) \dots \pi(x_{|x|})}$$

$$\beta(w) = \pi(y_1)\pi(y_2) \dots \pi(y_{|y|})\underline{\pi(x_1)\pi(x_2) \dots \pi(x_{|x|})}\pi(z_1)\pi(z_2) \dots \pi(z_{|z|})$$

It follows that $sub(\beta(x), \beta(w))$ is true.

**The Only If Case**: $sub(\beta(x), \beta(w)) \rightarrow sub(x, w)$

$\pi^{-1}$ is a permutation on Σ. By Corollary 3, $\beta^{-1}$ is the string substitution for $\pi^{-1}$. Therefore, the result of The If Case above applies, so that

$$sub(\beta(x), \beta(w)) \rightarrow sub\left(\beta^{-1}(\beta(x)), \beta^{-1}(\beta(w))\right)$$

By Corollary 1, $\beta^{-1}(\beta(x)) = x$, and $\beta^{-1}(\beta(w)) = w$. It follows that $sub(x, w)$ is true.

**End Proof**

### *7.2 Extending String Substitutions to Stringsets and Strictly Local Grammars*

#### *7.2.1 Transformations of Stringsets*

A **stringset** $L$ is simply a set of strings on some alphabet Σ: $L \subseteq \Sigma^*$

The transformation of a stringset is the result of applying the same string substitution to every string in the stringset. Given a stringset $L$ and a string substitution $\beta$, the **stringset transformation for $\boldsymbol{\beta}$** of $L$, $t_{lang}: \mathcal{P}(\Sigma^*) \rightarrow \mathcal{P}(\Sigma^*)$, is

$$t_{lang}(L) = \{\beta(w) | w \in L\}$$

#### *7.2.2 Definition of Strictly Local Grammars*

This section provides a formal definition of a strictly local (SL) grammar, and then defines the transformation of such a grammar, based on $\pi: \Sigma \rightarrow \Sigma$ above. The definition of an SL grammar provided here is derived from the one given in (Rogers et al, 2013), but differs in several respects. A grammar is expressed as a set of negative k-factors, where a string is admitted by the grammar if and only if the string does not contain any of the grammar's k-factors as a substring. It shares with (Rogers et al 2013) the use of the string delimiter symbols[9] ⋊ and ⋉ in the k-factors.

Given a string $w$, we convert it to a delimiter-enhanced string by adding the string delimiter symbols, using function $d(w)$.

$$d(w) = \rtimes w \ltimes$$

This is the strictly local version of a parsed string (analogous to a tree); grammars evaluate these delimiter-enhanced strings. A delimiter-enhanced string is itself a string on the enhanced alphabet Δ.

$$\Delta = \Sigma \cup \{\rtimes, \ltimes\}$$

Note that $|d(w)| = |w| + 2$.

The set of **canonical k-factors** on Σ, denoted $\Gamma_k$, is the union of the following four sets (each of which is necessarily finite):

[9] "string delimiter symbol" here means exactly the same thing as "string boundary symbol" earlier in the paper.

$$\{\rtimes x \ltimes | x \in \Sigma^i, 0 \leq i \leq (k-2)\}$$
$$\{\rtimes x | x \in \Sigma^{k-1}\}$$
$$\{x \ltimes | x \in \Sigma^{k-1}\}$$
$$\{x | x \in \Sigma^k\}$$

A **strictly k-local ($SL_k$) grammar $G$** is any subset of the set of canonical k-factors on alphabet Σ.

$$G \subseteq \Gamma_k$$

We will say that string $w \in \Sigma^*$ satisfies $SL_k$ grammar $G$ if and only if none of the k-factors in $G$ is a substring of the delimiter-enhanced string $d(w)$. Let $w \vDash G$ denote the circumstance that string $w$ satisfies $G$.

$$w \vDash G \leftrightarrow \left(\forall f \in G, \neg sub\left(f, d(w)\right)\right)$$

The stringset described by $G$, $strset(G)$, is the set of all strings that satisfy $G$.

$$strset(G) = \{w | w \in \Sigma^*, w \vDash G\}$$

*7.2.3 Transformations of SL Grammars*

Let $\pi: \Sigma \to \Sigma$ be a bijection on alphabet Σ. Let $\Delta = \Sigma \cup \{\rtimes, \ltimes\}$ be the enhanced alphabet for Σ. We extend the permutation $\pi$ to the enhanced alphabet via the bijection $\pi_\Delta(b): \Delta \to \Delta$.

$$\pi_\Delta(b) = \begin{cases} \pi(b) & \text{if } b \in \Sigma \\ \rtimes & \text{if } b = \rtimes \\ \ltimes & \text{if } b = \ltimes \end{cases}$$

The permutation $\pi_\Delta$ maintains permutation $\pi$ while leaving the delimiter symbols unchanged (each one substitutes for itself). We can extend this alphabet permutation to a string substitution just as we did for $\pi$ earlier. Let $\beta_\Delta: \Delta^* \to \Delta^*$ be defined as

$$\beta_\Delta(w) = \begin{cases} \lambda & \text{if } w = \lambda \text{ (the empty string)} \\ \pi_\Delta(w_1)\pi_\Delta(w_2) \ldots \pi_\Delta\left(w_{|w|}\right) & \text{if } w \neq \lambda \end{cases}$$

Theorem 2 applies to $\beta_\Delta$ for $\pi_\Delta$ just as it did to $\beta$ for $\pi$, so $\beta_\Delta$ is a bijection and thus the string substitution for $\pi_\Delta$.

We can use $\beta_\Delta$ to define a transformation of an $SL_k$ grammar $G$ by applying $\beta_\Delta$ to all of the k-factors of $G$: a transformed grammar $t_{gram}(G)$ is the set containing the string substitution of each of the k-factors in $G$. Let the **grammar transformation for an $SL_k$ grammar**, $t_{gram}: \mathcal{P}(\Gamma_k) \to \mathcal{P}(\Gamma_k)$, be defined as

$$t_{gram}(G) = \{\beta_\Delta(f) | f \in G\}$$

Although $\beta_\Delta$ is defined over $\Delta^*$, our concern is the string substitution of k-factors in $\Gamma_k$. We need to prove that $\Gamma_k$ is closed under $\beta_\Delta$, that is, that $(\forall f \in \Gamma_k)(\beta_\Delta(f) \in \Gamma_k)$.

**Lemma 1**: If $x \in \Sigma^j$, then $\beta_\Delta(x) = \beta(x)$, and $\beta_\Delta(x) \in \Sigma^j$.
**Proof**
String substitution $\beta_\Delta(x)$ applies alphabet permutation $\pi_\Delta$ to each symbol of $x$. If $x \in \Sigma^j$, then $\forall x_i (x_i \in \Sigma)$, and therefore $\forall x_i \left(\pi_\Delta(x_i) = \pi(x_i)\right)$. Thus, $\beta_\Delta(x) = \beta(x)$.
It follows from the definition of $\beta$ that $\beta$ is length-preserving, so $x \in \Sigma^j$ entails that $\beta(x) \in \Sigma^j$. Since $\beta_\Delta(x) = \beta(x)$, $\beta_\Delta(x) \in \Sigma^j$.
**End Proof**

**Theorem 4**: let $\beta_\Delta$ be as defined above. Then $\Gamma_k$ is closed under $\beta_\Delta$.
**Proof**
$\Gamma_k$ is the union of four disjoint sets. We show that each of these four sets is itself closed under $\beta_\Delta$.
Set 1: $\{\rtimes x \ltimes | x \in \Sigma^i, 0 \leq i \leq (k-2)\}$.
$\beta_\Delta$ leaves the delimiters in place, so $\beta_\Delta(\rtimes x \ltimes) = \rtimes \beta_\Delta(x) \ltimes$. By Lemma 1, $\beta_\Delta(x) \in \Sigma^i$, so $\beta_\Delta(\rtimes x \ltimes)$ is a member of Set 1.
Set 2: $\{\rtimes x | x \in \Sigma^{k-1}\}$
$\beta_\Delta$ leaves the delimiters in place, so $\beta_\Delta(\rtimes x) = \rtimes \beta_\Delta(x)$. By Lemma 1, $\beta_\Delta(x) \in \Sigma^{k-1}$, so $\beta_\Delta(\rtimes x)$ is a member of Set 2.
Set 3: $\{x \ltimes | x \in \Sigma^{k-1}\}$
$\beta_\Delta$ leaves the delimiters in place, so $\beta_\Delta(x \ltimes) = \beta_\Delta(x) \ltimes$. By Lemma 1, $\beta_\Delta(x) \in \Sigma^{k-1}$, so $\beta_\Delta(x \ltimes)$ is a member of Set 3.
Set 4: $\{x | x \in \Sigma^k\}$
By Lemma 1, $\beta_\Delta(x) \in \Sigma^k$, so $\beta_\Delta(x)$ is a member of Set 4.
Because $\Gamma_k$ is the union of Sets 1-4, $(\forall f \in \Gamma_k)(\beta_\Delta(f) \in \Gamma_k)$. Thus, $\Gamma_k$ is closed under $\beta_\Delta$.
**End Proof**

*7.3 Structure Preservation in Transformed SL Grammars*

**Theorem 5**: Let $w \in \Sigma^*$, and $G$ an SL grammar on Σ. Then

$$w \vDash G \leftrightarrow \beta(w) \vDash t_{gram}(G)$$

**Proof**
By Lemma 1, for any $w \in \Sigma^*$, $\beta_\Delta(w) = \beta(w)$.
Note that for any $w \in \Sigma^*$, $\beta_\Delta\big(d(w)\big) = d\big(\beta_\Delta(w)\big)$, as $d$ inserts delimiters, while $\beta_\Delta$ leaves pre-existing delimiters in place if they are present, and does not insert delimiters.
**The If Case**: $w \vDash G \rightarrow \beta(w) \vDash t_{gram}(G)$
Suppose, to the contrary, that $\beta(w) \nvDash t_{gram}(G)$, and thus that $\beta_\Delta(w) \nvDash t_{gram}(G)$. Then $\exists f \in G$ such that $sub\left(\beta_\Delta(f), d\big(\beta_\Delta(w)\big)\right)$, and thus that $sub\left(\beta_\Delta(f), \beta_\Delta\big(d(w)\big)\right)$. $\beta_\Delta$ is the string substitution for $\pi_\Delta$, so by Corollary 3 $\beta_\Delta^{-1}$ is the string substitution for $\pi_\Delta^{-1}$, and by Theorem 3 $\beta_\Delta^{-1}$ is substring-preserving. Therefore, $sub\left(\beta_\Delta^{-1}\big(\beta_\Delta(f)\big), \beta_\Delta^{-1}\left(\beta_\Delta\big(d(w)\big)\right)\right)$. By Corollary 1, $sub\big(f, d(w)\big)$, which contradicts the premise that $w \vDash G$.
**The Only If Case**: $\beta(w) \vDash t_{gram}(G) \rightarrow w \vDash G$
Suppose, to the contrary, that $w \nvDash G$. Then $\exists f \in G$ such that $sub\big(f, d(w)\big)$. Because $\beta_\Delta$ is substring-preserving, it is also true that $sub\left(\beta_\Delta(f), \beta_\Delta\big(d(w)\big)\right)$, which contradicts the premise that $\beta(w) \vDash t_{gram}(G)$.
**End Proof**

Theorem 5 allows us to prove that, for any permutation $\pi: \Sigma \rightarrow \Sigma$, the transformation of grammar $G$, $t_{gram}(G)$, describes the transformation of the stringset for $G$, $t_{lang}\big(strset(G)\big)$.

**Theorem 6**: for any permutation $\pi: \Sigma \to \Sigma$, $strset\left(t_{gram}(G)\right) = t_{lang}\left(strset(G)\right)$.

**Proof**

**Case 1**: $\left[x \in t_{lang}\left(strset(G)\right)\right] \to \left[x \in strset\left(t_{gram}(G)\right)\right]$

Let $x \in t_{lang}\left(strset(G)\right)$.

By the definition of $t_{lang}$, $\beta^{-1}(x) \in strset(G)$, therefore $\beta^{-1}(x) \vDash G$.

By Theorem 5, $[\beta^{-1}(x) \vDash G] \to \left[\beta\left(\beta^{-1}(x)\right) \vDash t_{gram}(G)\right]$.

Composing inverses yields $\beta\left(\beta^{-1}(x)\right) = x$, so $x \vDash t_{gram}(G)$, and $x \in strset\left(t_{gram}(G)\right)$.

Thus, $\left[x \in t_{lang}\left(strset(G)\right)\right] \to \left[x \in strset\left(t_{gram}(G)\right)\right]$.

**Case 2**: $\left[x \in strset\left(t_{gram}(G)\right)\right] \to \left[x \in t_{lang}\left(strset(G)\right)\right]$

Let $x \in strset\left(t_{gram}(G)\right)$.

By the definition of $strset$, $x \vDash t_{gram}(G)$.

By Theorem 5, $\left[x \vDash t_{gram}(G)\right] \to [\beta^{-1}(x) \vDash G]$.

Then $\beta\left(\beta^{-1}(x)\right) \in t_{lang}\left(strset(G)\right)$.

Composing inverses yields $\beta\left(\beta^{-1}(x)\right) = x$, so $x \in t_{lang}\left(strset(G)\right)$.

Thus, $\left[x \in strset\left(t_{gram}(G)\right)\right] \to \left[x \in t_{lang}\left(strset(G)\right)\right]$.

Case 1 demonstrates that $t_{lang}\left(strset(G)\right) \subseteq strset\left(t_{gram}(G)\right)$, and Case 2 demonstrates that $strset\left(t_{gram}(G)\right) \subseteq t_{lang}\left(strset(G)\right)$. Therefore, $strset\left(t_{gram}(G)\right) = t_{lang}\left(strset(G)\right)$.

**End Proof**

*8. Interim Summary and Discussion*

We have presented a method for exploring the predictions of subregular phonology. The method involves a partial derangement, or permutation of an alphabet, a transformation of a grammar based on the partial derangement, and a transformation of the stringset produced by the grammar based on the partial derangement. The method allows the analyst to, given a pattern belonging to a computationally defined class, easily generate a collection of other patterns that necessarily also belong to that class.

Next, we examined the results of applying the method to particular seed patterns as represented by particular alphabets. We first applied the method to the Maranungku stress pattern represented by a two-symbol alphabet, Σ = {σ̊, σ}, and then a three-symbol alphabet, Σ = {σ́, σ̀, σ}. The result in the former case was two distinct patterns, one identical to the original seed pattern and one unattested. The result in the latter case was six distinct patterns, one identical to the original seed pattern and five unattested. We then applied the method to both the Maranungku stress pattern and the Wergaia stress pattern represented by a six-symbol alphabet, Σ = {Ĺ, Ĺ, Ȟ, L̀, H, L}, and then a twelve-symbol alphabet, Σ = {V́, D́, Ć, Ĺ, V̀, D̀, C̀, L̀, V, D, C, L}. Using the six-symbol alphabet, derangement of the Maranungku pattern results in 90 distinct stress patterns, and derangement of the Wergaia pattern results in 720 distinct stress patterns. Given the number of patterns produced, we were unable to discuss them all, but we did discuss a handful of patterns to illustrate the implausible types of quantity sensitivity and quantity restrictedness that permeate the deranged patterns. Using the twelve-symbol alphabet,

derangement of the Maranungku pattern results in 83,160 distinct patterns, and derangement of the Wergaia pattern results in 39,916,800 distinct patterns. Given the volume of patterns produced, we discussed a few of the types of patterns that arise only in the most general terms and merely highlighted some of the problematic phenomena predicted.

When the number of deranged patterns for a given seed pattern and alphabet was small enough to make an informal assessment feasible, we considered whether the number of fixed points in a partial derangement was related to the plausibility of the pattern resulting from derangement. It was not. A greater degree of symbol scrambling did not necessarily result in a higher level of implausibility. The plausibility of a pattern depends primarily on the content of a symbol—its substance—and the appropriateness of that content occurring in the positions to which it was assigned, both in isolation and in combination with the content from other symbols. Even minimal scrambling was sufficient to locate symbols in problematic positions, both in isolation and in inappropriate combinations.

Finally, we provided a proof that a deranged strictly local grammar generates the analogously deranged stringset. This result validates the derangement method: the use of deranged stringsets as members of the typology predicted by the class of strictly local grammars with the same alphabet and *k*-value as the original grammar. The transformed grammar, and thus the transformed stringset, have the same computational properties as the pre-transformation originals. Further below, we provide a proof of the analogous result for the class of first-order logic grammars, which are capable of describing the class of all subregular languages.

Before turning to the more general proof, however, we discuss the significance of the results already presented for the relationship between the attested typology of stress patterns and the computational classes. We will then offer some preliminary thoughts on options for making theories based on subregular grammars more restrictive, and then we will address the significance of the choice of particular alphabets in subregular analyses.

### *8.1 Phonology and Computational Classes*

Typological accuracy is of central importance in judging the success of theories individually and in determining preferences among competing theories. A theory must be expressive enough to predict attested patterns, but it must also be restrictive enough to avoid unattested patterns. Typological accuracy has long been a central concern of phonology, and Subregular Phonology is no exception. Heinz (2018) asserts "… a theory not only needs to explain WHAT THERE IS, BUT ALSO WHAT THERE IS NOT." (Capitalization is original.) The results of the examination presented above indicate that the strictly local class is wildly unrestrictive. Attested phonological patterns make up a tiny fraction of the phonological patterns predicted by strict locality. Since most of the other subregular classes contain the patterns of the strictly local class, this means that most of the other subregular classes are wildly unrestrictive in the same way, as well.

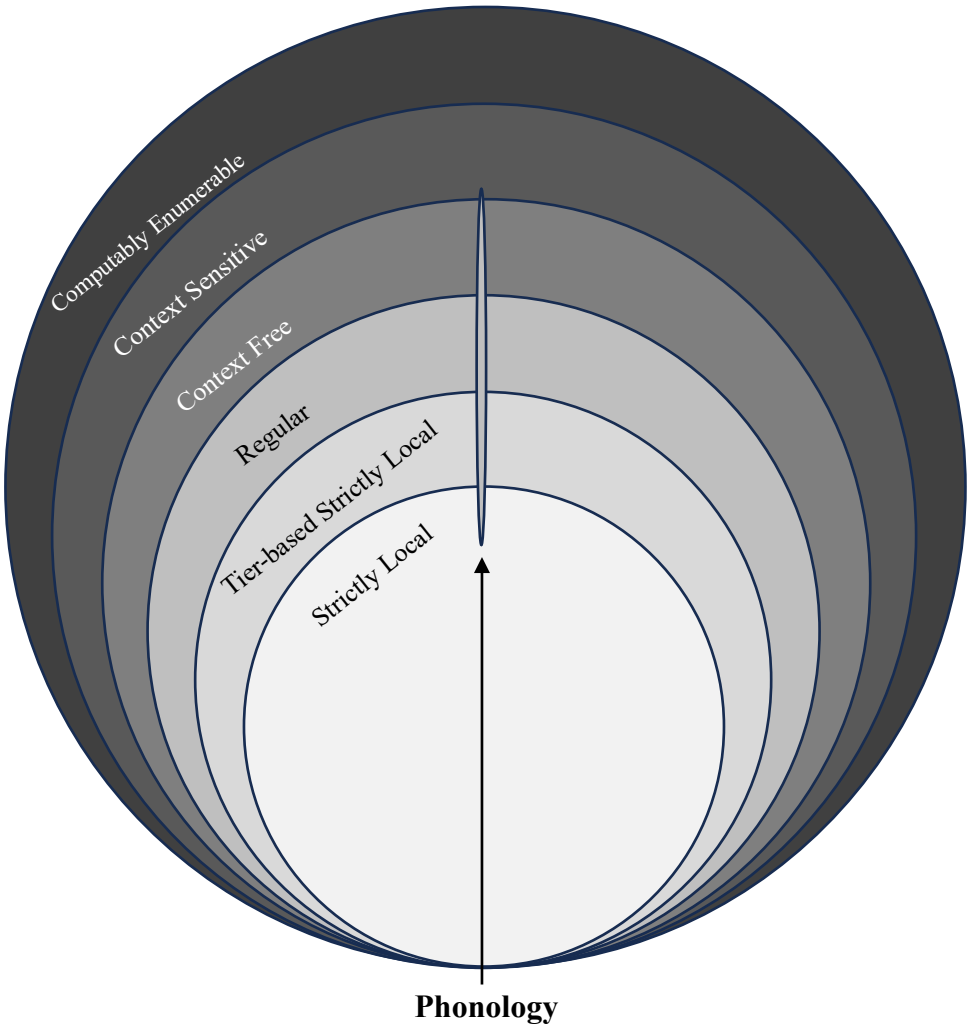
While this paper has focused on subregular phonology's restrictive adequacy, it is necessary to briefly discuss two claims that bear on its expressive adequacy in order to situate the attested typology of phonological patterns relative to the subregular classes. First, Heinz and Idsardi (2011, 2013) have claimed that phonology is about words and not about phrases or sentences. If true, this would absolve phonology of accounting for a wide range of phenomena that it has traditionally been responsible for. These phenomena include sandhi phenomena, phrasal stress, rhythm rule adjustments, intonation, many other types of tonal phenomena, and prosodic domains and boundaries. Second, some phonologists have proposed to exclude even

some word-level phenomena from the domain of phonology, proposing that reduplication is a morphological or syntactic process rather than a phonological process (Inkelas and Zoll, 2005; Roark and Sproat 2007; Heinz and Idsardi 2013; Chandlee 2014; Graf 2022). This would absolve phonology of accounting for a phenomenon that appears to fall outside the expressive capabilities of subregular—and even regular—grammars.[10] We reject both claims. Reduplication, phrasal stress, rhythm rule adjustments, intonation, prosody, etc. all primarily concern the distribution of linguistically relevant sounds, and they all properly fall within the domain of phonology.

The illustration in (64) provides a very rough picture reflecting our view of the range of attested phonological patterns relative to the domains of various computational classes. The set of attested phonological patterns lies within the tiny sliver that slices across the boundaries of multiple computational classes, but the sliver only includes a small slice of even the most restrictive class. The illustration is not exact, of course. Our view is that reduplication drags phonology outside of the purely context-free; we have no opinion at this stage as to whether phonology extends outside of the purely context-sensitive. The point of the illustration is our view that phonology simultaneously extends well beyond even the class of regular languages, while simultaneously being only a small part of any computational class.

[10] Certain types of word stress patterns are also beyond the descriptive capacity of subregular—and even regular—grammars, though Graf (2010, 2022) and others have expressed doubts about the accuracy of the patterns' descriptions.

(64)




Since none of the computational classes is even a remotely close match for the attested typology of phonological patterns, it should be clear that it is impossible to accurately predict the phonological typology with any particular computational class.

*8.2 Characteristic Properties of Stress Patterns*

Stress and accent patterns have a number of well-established characteristics, and one of the priorities for phonologists working on stress and accent is to account for these characteristics. Every word has exactly one main stress, for example. Long lapse sequences over light syllables are possible, but long clash sequences over light syllables are not possible. When clash occurs in forms without heavy syllables, it occurs near edges. Stress can be attracted to heavy syllables at the expense of light syllables, but stress cannot be attracted to light syllables at the expense of heavy syllables. Many others might be listed.

Heinz (2018) favorably contrasts the typological predictions of subregular phonology to those of Optimality Theory (OT; Prince and Smolensky 1993/2004), but he does so without

actually exploring the predictions of existing theories in either framework. Optimality Theory has provided a basis for several general accounts of stress and accent, focused mostly on quantity-insensitive patterns. Among these are the Weak Layering Generalized Alignment approach of McCarthy and Prince (1993), Crowhurst and Hewitt (1995), and Alber (2005), among others; the grid-based approach of Gordon (2002); the Weak Bracketing approach of Hyde (2002, 2016); and the Internally Layered Foot approach of Martinez-Paricio and Kager (2015). Much of the OT literature on stress and accent has been devoted to what—in isolation—appear to be significant differences in the typological predictions of these major proposals. Set against the backdrop of the predictions of subregular phonology, however, there really are no major differences in the predictions of the OT approaches. Against this backdrop, the major OT approaches are distinguished from each other only by the slightest variation in their very high degree of typological accuracy. None of the OT approaches overgenerates at nearly the level of even the class of strictly local grammars whose *k*-value is two. None of them predicts strings where a single unstressed syllable is preceded or followed by a string of syllables with alternating primary and secondary stress. None of them predicts avoidance of stress on heavy syllables in favor of light syllables. None of them predicts quantity sensitivity in odd-numbered syllables only or in even-numbered syllables only. None of them predicts that even-numbered positions or odd-numbered positions can be restricted to syllables of a certain weight while leaving their stress level optional. None of them predicts languages that do not allow light syllables. None of them predicts languages that do not allow stressless syllables.

### *8.3 Externally Imposed Restrictiveness*

Based on work by Edlefsen et al (2008), Rogers et al (2013) note that 81 of the 109 stress patterns in Heinz's (2007, 2009) database are Strictly Local. All 81 have a *k*-value where $k \leq 6$, and 53 have a *k*-value where $k \leq 3$. They take this as an indication of the simplicity of most stress patterns. Simplicity is not the same thing as restrictiveness, however. In Section 5, we saw how the derangement of the Maranungku pattern resulted in 83,160 phonologically distinct stress patterns. The Maranungku pattern has a *k*-value of two, but so does each of the 83,160 phonologically distinct derangements. We also saw in Section 6 how the derangement of the Wergaia pattern resulted in 39,916,800 phonologically distinct stress patterns. The Wergaia pattern has a *k*-value of three, but so do each of the 39,916,800 phonologically distinct derangements. Every one of the 53 attested patterns with a *k*-value of $k \leq 3$ would yield its own set of distinct derangements, making the number of distinct patterns much larger than the number resulting from just Maranungku and Wergaia alone.[11] Even at a *k*-value of $k \leq 3$, then, the number of distinct derangements of attested patterns is many orders of magnitude larger than the set of attested patterns themselves.

Given these results, it is clear that in order for subregularity to be the right theory of phonology, a way must be found to blunt the power of subregularity! The most convenient way to do this, of course, would be to shift responsibility for restrictiveness to something external to the phonological grammar, perhaps to phonetics. Stress and accent patterns, however, are not shaped by phonetic constraints so much as they are shaped by rhythmic and metrical constraints, constraints that are found at work beyond the domain of speech sounds. One might propose that

[11] As there are likely other patterns that are relationally distinct from any of the attested patterns, the number of distinct stress patterns with a *k*-value of $k \leq 3$ overall is likely much larger still.

the influence of rhythmic and metrical constraints is also external and that they do not need to be incorporated directly into the phonological grammar. Another potential source of restrictiveness that might be appealed to are functional considerations.[12] If these were also external, there would be no need to incorporate them into the phonology. There are likely other sources of restrictiveness that might be appealed to, as well.

The problem with the external approach is that phonetic, metrical, rhythmic, and functional pressures are universal but they are reflected in phonological patterns in a wide variety of ways. This is due at least partially to the fact that—while universal—phonetic, metrical, rhythmic, and functional considerations are also often in conflict, and different languages can resolve the conflicts in different ways. While one might take the position that the mechanisms that resolve these conflicts in the ways that yield this variety are external to phonology, there seems to be nothing beyond the desire to keep phonology itself computationally simple to support it. The result would simply be a theory in which the vast majority of the explanation for phonological phenomena lies "outside" of phonology itself. It seems to us much more likely that it is one the roles of phonology to manage the conflicts between the different phonetic, metrical, rhythmic and functional considerations that affect the distribution of linguistically relevant sounds.

That leaves the internal approach: incorporating within phonological theory principles which have the effect of greatly restricting the purely computational typology in a way that steers the typology much closer to the attested patterns. Given the vast overgeneration of strictly local grammars, the additional principles will end up carrying a substantial portion of the explanatory load.

### *8.4 The Significance of the Choice of Alphabets*

In the key works assessing which stress patterns can be implemented with strictly local grammars, only Rogers and Lambert (2019) discuss the alphabet assumed for the assessment. Rogers and Lambert assume an alphabet with 15 symbols representing combinations of three degrees of stress and five degrees of syllable weight. Given the results of the other key works, those seem also to have employed symbols representing syllables for the representation of stress patterns. The choice of the alphabet matters for at least four reasons.

First, as we have seen in the preceding sections, the number of partial derangements that are possible increases factorially as the size of the alphabet increases, and this establishes the upper limit on the number of distinct grammars and languages that can result from derangement of a seed pattern. The largest alphabet considered above had 12 symbols, yielding 12!, or 479,001,600, partial derangements. Rogers and Lambert's alphabet of 15 symbols yields 15!, or 1,307,674,368,000, partial derangements.

Second, the way that a given alphabet is used to represent a stress pattern is also significant. If Rogers and Lambert only used one of their five degrees of syllable weight to represent Maranugku, for example, since Maranungku is quantity insensitive, then the number of unique patterns yielded by derangement would be 2,730, a number that is still more than 20 times the number of attested patterns. If, however, they used two of their assumed five degrees of

[12] Heinz and Idsardi (2013) claim that phonology is atelic—that it does not have a purpose or a function. For those who take this position, phonological typologies could not be shaped by functional considerations either internally or externally. The claim is straightforwardly false, however. See Trubetskoy 1969, Kaye 1989, and Hyde 2025, among others, for discussion of the functions of phonology.

syllable weight to represent Maranungku, since Maranungku has syllables of a type that are potentially heavy as well as syllables that are always light, then the number of unique patterns yielded by derangement would grow to 450,450. In the former scenario, Rogers and Lambert would be stipulating quantity insensitivity in the representation of Maranungku, a dubious choice. In the latter scenario, they would be deriving quantity insensitivity through the mechanisms of the grammar, a sounder choice.

Third, the alphabet may or may not incorporate the particular characteristics that actually create weight distinctions—such as a syllable being open, being closed, containing a diphthong, or containing a long vowel—as we incorporated them in Sections 5 and 6. The alternative to incorporating these weight-creating characteristics into the alphabet is to stipulate abstract stress levels, as with the six-symbol H- and L-based alphabet employed in Sections 3 and 4 or the five-level weight distinction that Rogers and Lambert appear to have employed. With alphabets that incorporate weight creating characteristics, patterns with two-way weight distinctions based on CV, CVC, and CV: syllables, for example, can differ in whether CVC syllables pattern with CV: syllables in counting as heavy or pattern with CV syllables in counting as light. Alphabets like the H- and L-based alphabet employed in Sections 3 and 4 and the alphabet likely employed by Rogers and Lambert miss these distinctions.

Weight creating characteristics may also have positional restrictions that are independent or semi-independent of the stress pattern, as we saw in the case of Wergaia. In the Wergaia case, the choice of alphabet affected the number of distinct patterns created by derangement using Wergaia as the seed pattern, but it did not affect the *k*-value of Wergaia and, therefore, did not affect the *k*-value of the deranged patterns. It is possible, however, that the choice of alphabet would have affected the *k*-value of both seed pattern and deranged patterns had a different seed pattern been chosen.

Finally, the alphabet chosen to represent a stress pattern is a key factor in assessing the computational power required to produce the stress pattern. It is likely that all analyses of stress patterns in terms of strictly local grammars have used symbols that represent stress patterns at the syllable level, like the analysis of Rogers and Lambert and like the analyses considered above, rather than symbols that represent the stress patterns at the segmental level. This choice allows subregular phonology to benefit from hierarchical phonological structure without paying the price for constructing hierarchical phonological structure. In the long run, the issue of the construction of hierarchical phonological structure will need to be dealt with. The syllable is a hierarchical structure connected to segments, and, as such, the construction of syllabic representation must be coordinated with the construction of segmental representation. This is not just a matter of how a theory groups segments together. Segmental details are crucial to determining the potential of a syllable to be treated as heavy, and stress patterns can compel changes and restrictions at the segmental level.

## *9. Proof: Transformed FOL Grammars Yield Transformed Stringsets*

### *9.1 The Subregular Hierarchy for Stringsets*

The term “subregular hierarchy” refers to subset relations between the subregular classes of stringsets. The top class of formal stringsets in the subregular hierarchy is the NonCounting stringsets, or NC (McNaughton & Papert 1971). Every other class in the subregular hierarchy is a subset of NC. NC is notable in that it is co-extensive with the class of formal stringsets

definable by first-order logic (FOL)[13]. Any stringset that is in NC is described by some sentence of FOL. Importantly, any stringset that can be described by a sentence of FOL is in NC.

### *9.2 Logical Descriptions of Stringsets*

The logical description of stringsets presented in this section is derived largely from the one given by Ebbinghaus and Flum (2006, pp. 108-117).

Logical descriptions of strings are understood to have, as their universe of entities, positions in a string. They use unary predicates to represent the assignment of a specific symbol to a specific position, and a binary predicate to represent the ordering among the positions. For descriptions of strings on the alphabet Σ, the logic includes

- $<$ is a binary predicate expressing an antireflexive total order relation on the positions.
- $\{P_a | a \in \Sigma\}$ are unary predicates, one for each symbol of the alphabet, indicating the positions at which each symbol occurs.

#### *9.2.1 The Vocabulary*

A **vocabulary** $\tau$ is a finite set of relation symbols and constant symbols. The vocabulary assigns an arity of 1 or more to each relation symbol. The vocabulary serves as a kind of interface between a logic and possible models for sentences of that logic. It defines the symbols that appear both as predicates in the logic and as relations in the models, with each relation of a model providing the extension for the predicate of the same name in the logic.

For logic sentences describing stringsets, the vocabulary will always have the binary predicate $<$ for the order on the positions. It will also have a unary predicate $P_a$ for each symbol $a$ of the alphabet. Thus, the vocabulary will be $\tau(\Sigma) = \{<\} \cup \{P_a | a \in \Sigma\}$.

For stringsets based on the alphabet $\Sigma = \{j, k\}$, the corresponding vocabulary is

$$\tau(\Sigma) = \{<, P_j, P_k\}$$

Of interest here is a variant of FOL that includes the equality symbol '='. The semantic sense of '=' is assumed not to vary, so '=' is classified as a logical symbol rather than as a relational one, and it is not included in the vocabulary (Ebbinghaus & Flum 2006, p. 4; Enderton 2001, p. 70; Hodges 1993, p. 12).

Given alphabet Σ, the set of FOL formulas for vocabulary $\tau(\Sigma)$ is denoted FOL$[\tau(\Sigma)]$, typically FOL$[\tau]$ when Σ is apparent from context.

#### *9.2.2 Baseline Conditions for Strings*

To fully describe strings, a logic sentence must include the conditions that must be satisfied to qualify as a string at all. There are three core conditions:

- the relation $<$ must be an antireflexive total ordering.
- every position must be assigned an alphabet symbol.
- no position may be assigned more than one alphabet symbol.

[13] First order logic with equality, to be a bit more precise.

Each of those conditions can be expressed with first-order sentences.
Antireflexive total ordering:

| | |
|---|---|
| $\forall x \neg (x < x)$ | antireflexive |
| $\forall x \forall y (x < y \vee y < x \vee x = y)$ | antisymmetric |
| $\forall x \forall y \forall z \big( (x < y \wedge y < z) \rightarrow x < z \big)$ | transitive |

Required alphabet symbol:

$$\forall x \bigvee_{a \in \Sigma} P_a(x)$$

Unique alphabet symbol:

$$\bigwedge_{\substack{a,b \in \Sigma \\ a \neq b}} \forall x \neg \big( P_a(x) \wedge P_b(x) \big)$$

The "quantified" disjunction and conjunction operators are meta-expressions, abbreviations for disjunctions/conjunctions of finite numbers of formulas. They are essentially templates for creating actual FOL$[\tau(\Sigma)]$ sentences. If the alphabet $\Sigma = \{j, k, m\}$ were in use, the actual FOL$[\tau(\Sigma)]$ sentence for requiring that each position have an alphabet symbol would be

$$\forall x \left( P_j(x) \vee P_k(x) \vee P_m(x) \right)$$

and the actual FOL$[\tau(\Sigma)]$ sentence for requiring that each position have a unique alphabet symbol would be

$$\Big( \forall x \neg \left( P_j(x) \wedge P_k(x) \right) \Big) \wedge \Big( \forall x \neg \left( P_j(x) \wedge P_m(x) \right) \Big) \wedge \Big( \forall x \neg \big( P_k(x) \wedge P_m(x) \big) \Big)$$

We can conjoin all these sentences into one larger sentence, and for convenience give that combined sentence the label $\varphi_{str}$.

$$\varphi_{str} \coloneqq [\forall x \neg (x < x)] \wedge [\forall x \forall y (x < y \vee y < x \vee x = y)] \wedge \big[ \forall x \forall y \forall z \big( (x < y \wedge y < z) \rightarrow x < z \big) \big] \wedge \left[ \forall x \bigvee_{a \in \Sigma} P_a(x) \right] \wedge \left[ \bigwedge_{\substack{a,b \in \Sigma \\ a \neq b}} \forall x \neg \big( P_a(x) \wedge P_b(x) \big) \right]$$

The stringset described by $\varphi_{str}$ is the set of all strings formable on the alphabet Σ. To describe some other stringset, we would conjoin $\varphi_{str}$ with additional sentences (within the same vocabulary) to form a more restrictive description.
The sentence $\varphi_{str}$ is fixed for a given vocabulary. The only thing in the convenience form shown above that can vary is the alphabet symbol Σ, and that only appears in the quantification expressions of the meta-expression operators. The alphabet is implicitly contained in the vocabulary: for each symbol predicate $P_a$ in the vocabulary, there is a distinct symbol $a$ in Σ. Once the vocabulary is selected, the exact actual sentence for $\varphi_{str}$ is fully determined.

### *9.3 Models for Strings*

Interpretations of FOL sentences are understood in terms of model theory. The formal structures most commonly used to realize strings are conventionally referred to as **word models**, in

keeping with a tendency to use "word" as a synonym for "string" (e.g., Ebbinghaus & Flum 2006, p. 105). To a linguist, this may be somewhat misleading, as strings are widely used in many linguistic domains. Just as phonology is concerned with linguistic structures both larger than and smaller than words, "word models" are employed to model linguistic structures both larger and smaller than words.

### *9.3.1 Word Models*

First-order structures are defined over vocabularies, just as first-order sentences are. For strings over alphabet Σ, the vocabulary will be $\tau(\Sigma) = \{<\} \cup \{P_a | a \in \Sigma\}$, the same vocabulary as for sentences used to describe stringsets.

A **word model** is a mathematical object $M = (D^M, <^M, (P_a^M)_{a\in\Sigma})$.

- $D^M$ is a non-empty finite domain of positions, one for each string position.
- $<^M \subseteq D^M \times D^M$ is a binary relation, indicating the order of occurrence of the positions.
- $(P_a^M)_{a\in\Sigma}$ are unary relations such that each $P_a^M \subseteq D^M$, with $P_a^M$ containing the positions at which $a$ occurs in the string.

The relations of a word model must match the vocabulary in use.

For every word model, the binary relation '=' is presumed to be defined as $\{(x, x) | x \in D\}$. This provides the extension for the predicate '=' in any model relative to that model's domain.

In principle, a string $w$ has numerous first-order models. However, any two models for the same string are isomorphic, so it is reasonable to refer to "the" model $M_w$ of string $w$ (Ebbinghaus & Flum 2006, p. 109).

### *9.3.2 Word Models are Non-Empty*

As a broad generalization, computer scientists like the empty string; they find it very useful. As another broad generalization, model theorists have a strong aversion to empty models (models whose domain is the empty set), and often explicitly exclude them from consideration (e.g., Ebbinghaus & Flum 2006, p. 1). Hodges (1993, p. 42) lists two "embarrassing consequences" of allowing empty models. The analogous word model for the empty string has no symbols and therefore no symbol positions, so it is indeed an empty model.

This dispreference for empty models is strong enough that it leads scholars like Ebbinghaus & Flum (2006, p. 114) to describe the class of stringsets definable in first-order logic as the *plus free regular* stringsets, which are the star free stringsets (aka NC) without the empty string (the role of Kleene star is instead filled by Kleene plus).

If contending with the empty string is required, there are workarounds, such as the use of boundary markers (like ⋊ and ⋉), at the cost of some additional complexity in the logical descriptions and the models. We will not concern ourselves further with the matter in this paper.

### *9.3.3 Word Models are Finite*

By definition, a string is finite. The size of a word model's domain directly reflects the length of the string. The definition of a word model explicitly restricts it to a finite domain, in deference to

the finiteness of the strings being modeled. This is not an empty concern: if a FOL sentence has arbitrarily large finite models, then it has an infinite model (Enderton 2001, p. 147). Thus, if a logic description of a stringset is satisfiable by models with arbitrarily long finite sequences of symbols, then it can be satisfied by some infinite sequence of symbols. It is those models of infinite symbol sequences that are disallowed by restricting word models to finite domains.

### *9.4 Describing Stringsets with First-Order Logic Sentences*

The connection between word models and logic sentences comes via the vocabulary. The binary predicate symbol $<$ in the logic has as its denotation in model $M$ the binary relation labeled $<^M$. Each of the unary predicates $P_a$ has as its denotation the unary relation labeled $P_a^M$.

#### *9.4.1 Dealing with the Successor Relation, Minimal Element, and Maximal Element*

Some classes in the subregular hierarchy, such as SL, are most succinctly described with the use of a successor relation for describing the relative order of the positions of a string. A successor relation holds only of two entities when one of them immediately follows the other, with no other entities ordered in-between. The distinction is between the total ordering relation $<$ and the successor relation $S$. If we take the usual ordering on integers for $\{1,2,3\}$, the total ordering $<$ of the set is

$$< = \{(1,2), (1,3), (2,3)\}$$

Note that $1 < 3$ is true for the total ordering, even though 1 and 3 are not consecutive in the order. By contrast, the successor relation $S$ on the same ordered set would be

$$S = \{(1,2), (2,3)\}$$

$S(1,3)$ is false, because 3 is not the successor of 1.

When describing SL stringsets using first-order logic sentences, the descriptive power of first-order logic is sufficient to derive the successor relation from the total ordering relation (Ebbinghaus & Flum 2006, p. 5). For readability, we will denote the successor relation with the substitution meta-expression $\psi_{succ}(x, y)$, read "the successor of $x$ is $y$":

$$\psi_{succ}(x, y) = x < y \wedge \forall z\big(x < z \rightarrow (y < z \vee y = z)\big)$$

It is also useful to be able to refer to the minimal and maximal elements of the string. The minimal element is the first position in the string (the position that appears immediately after the beginning-of-string delimiter $\rtimes$ in an SL grammar). The maximal element is the last position in the string (the position that appears immediately before the end-of-string delimiter $\ltimes$ in an SL grammar). The properties of minimal and maximal element can also be expressed in first-order logic, derived from the total order relation $<$, and for readability we will denote the properties with substitution meta-expressions as follows (Ebbinghaus & Flum 2006, p. 109):

$$\psi_{min}(x) \coloneqq \forall y \neg (y < x)$$
$$\psi_{max}(x) \coloneqq \forall y \neg (x < y)$$

The expressions $\psi_{succ}(x, y)$, $\psi_{min}(x)$ and $\psi_{max}(x)$ are substitution meta-expressions (van Dalen 2013, p. 63). They serve only to enhance readability. In the actual sentence, each occurrence of a meta-expression will have been substituted for by the corresponding formula on the right side of the definition, with the appropriate assignments to the arguments $x$ and $y$ substituted in. Thus, $\psi_{succ}$, $\psi_{min}$ and $\psi_{max}$ do not appear in the vocabulary.

*9.4.2 An FOL Grammar Defines a Set of Word Models*

Generally speaking, a grammar is a finite description of a stringset. Given a vocabulary $\tau$, an **FOL[$\tau$] grammar** is a sentence that conjoins $\varphi_{str}$ with zero or more other sentences. Let $G$ be such a sentence. The stringset described by $G$ consists of all the word models that satisfy $G$.

$$strset(G) \coloneqq \{M | M \vDash G\}$$

*9.4.3 Converting an SL Grammar into an FOL Grammar*

Any SL stringset can be described by some FOL grammar. The easy way to demonstrate this is by giving a recipe for converting an SL grammar into a an FOL grammar generating the same stringset (realized as a set of word models).

Suppose we have an $SL_2$ grammar $G_{SL} = \{ss, uu, \rtimes u\}$ on alphabet $\Sigma = \{u, s\}$. $G_{SL}$ describes strings in which stressed ($s$) and unstressed ($u$) syllables alternate, with stressed syllables in odd-numbered positions (from the left) of strings and unstressed syllables in even-numbered positions. Using vocabulary $\tau = \{<, P_s, P_u\}$, we can construct a corresponding FOL[$\tau$] grammar $G_{FOL}$ as follows.

The constraint (banned substring) $ss$ asserts that the substring $ss$ does not appear anywhere in any delimited string of the stringset. That constraint would be expressed in FOL[$\tau$] as

$$\neg(\exists\, x, y)[P_s(x) \wedge \psi_{succ}(x, y) \wedge P_s(y)]$$

The constraint $uu$ asserts that the substring $uu$ does not appear anywhere in any delimited string of the stringset. That constraint would be expressed in FOL[$\tau$] as

$$\neg(\exists\, x, y)[P_u(x) \wedge \psi_{succ}(x, y) \wedge P_u(y)]$$

The constraint $\rtimes u$ asserts that the substring $\rtimes u$ does not appear anywhere in any delimited string of the stringset. That is equivalent to saying that $u$ cannot be the first symbol of a non-delimited string. That constraint would be expressed in FOL[$\tau$] as

$$\neg(\exists\, x)[\psi_{min}(x) \wedge P_u(x)]$$

The full sentence for $G_{FOL}$ includes the conjunction of the sentences for the three constraints. In general, the logic sentence for an SL grammar will be the conjunction of the sentences for each of the constraints conjoined with the sentence $\varphi_{str}$. The overall conjunctive sentence is $G_{FOL}$.

$$G_{FOL} \coloneqq \varphi_{str} \wedge \neg(\exists\, x, y)[P_s(x) \wedge \psi_{succ}(x, y) \wedge P_s(y)] \wedge \neg(\exists\, x, y)[P_u(x) \wedge \psi_{succ}(x, y) \wedge P_u(y)] \wedge \neg(\exists\, x)[\psi_{min}(x) \wedge P_u(x)]$$

$\varphi_{str}$ is a sentence: there are no unbound variables. The same is true for the expressions for each of the constraints. Thus, $G_{FOL}$ is also a sentence. A word model on $\tau$ $M_w$ for any string $w$ can be evaluated for its satisfaction of $G_{FOL}$.

The model $Msus$ (for the string $sus$) would be the object $(D^{Msus}, <^{Msus}, P_s^{Msus}, P_u^{Msus})$, where

$$D^{Msus} = \{1,2,3\}$$
$$<^{Msus} = \{(1,2), (1,3), (2,3)\}$$
$$P_s^{Msus} = \{1,3\}$$
$$P_u^{Msus} = \{2\}$$

Model $Msus$ satisfies grammar $G_{FOL}$. The first constraint is

$$\neg(\exists\, x, y)[P_s(x) \wedge \psi_{succ}(x, y) \wedge P_s(y)]$$

There do not exist values contained in $D^{Msus}$ that can substitute for $x$ and $y$ and satisfy the formula $[P_s(x) \wedge \psi_{succ}(x,y) \wedge P_s(y)]$. If $1/x$, then $\psi_{succ}(x,y)$ could only be satisfied by $2/y$. But if $2/y$, $P_s(y)$ is not satisfied. Thus $1/x$ fails for any value of $y$. If $2/x$, then $P_s(x)$ is not satisfied. If $3/x$, then no $y$ will satisfy $\psi_{succ}(x,y)$. Thus, there do not exist values of $x$ and $y$ that can satisfy $[P_s(x) \wedge \psi_{succ}(x,y) \wedge P_s(y)]$, and the first constraint is satisfied. By similar reasoning, the other two constraints are also satisfied by $Msus$. $Msus$ also satisfies $\varphi_{str}$. Therefore, $Msus$ satisfies the sentence $G_{FOL}$, and is a string in the stringset described by $G_{FOL}$.

The model $Musu$ for the string $usu$ would be the object $(D^{Musu}, <^{Musu}, P_s^{Musu}, P_u^{Musu})$, where

$$D^{Musu} = \{1,2,3\}$$
$$<^{Musu} = \{(1,2),(1,3),(2,3)\}$$
$$P_s^{Musu} = \{2\}$$
$$P_u^{Musu} = \{1,3\}$$

Model $Musu$ does not satisfy grammar $G_{FOL}$. Specifically, it does not satisfy the third constraint:

$$\neg(\exists\, x)[\psi_{min}(x) \wedge P_u(x)]$$

If $1/x$, then $\psi_{min}(x)$ is satisfied (1 is the minimal element in the order) and $P_u(x)$ is satisfied. Thus, there does exist a value for $x$, namely $1/x$, such that $[\psi_{min}(x) \wedge P_u(x)]$ is satisfied, and thus the third constraint is **not** satisfied. Therefore, $Musu$ does not satisfy the sentence $G_{FOL}$, and is not in the stringset described by $G_{FOL}$.

### *9.5 Transforming FOL Grammars and Word Models*

For alphabet Σ, the vocabulary is $\tau = \{<\} \cup \{P_a | a \in \Sigma\}$. The corresponding logic is FOL$[\tau]$. An FOL$[\tau]$ grammar is any FOL grammar that is also a sentence of FOL$[\tau]$.

Let $G1$ be an FOL$[\tau]$ grammar, and let $L1$ be the stringset described by $G1$. Let $\pi$ be a permutation of Σ. Our goal is to prove that if we permute the position predicates $P_a$ in $G1$ following the direction of $\pi$, obtaining FOL$[\tau]$ grammar $G2$, then $G2$ describes the stringset $L2$ which is precisely the stringset transformation of $L1$ based on the string substitution for $\pi$. In other words, we will prove a result analogous to the one previously proven for SL grammars, this time for FOL grammars.

#### *9.5.1 Substituting Vocabulary Relation Symbols*

The proof given here makes use of the concept of **translations between definitionally equivalent logical theories**. This proof relies in particular on the presentation given by Hodges (1993, pp. 58-62).

Hodges' result of primary interest is that, if two theories $T_1$ and $T_2$ are definitionally equivalent, then any model $M_1$ of $T_1$ can be turned into a model $M_2$ of $T_2$, and that process can be inverted to turn $M_2$ of $T_2$ back into the original $M_1$ of $T_1$. In this situation, the models $M_1$ and $M_2$ are also said to be definitionally equivalent. The result is aimed at translating between theories with different vocabularies[14]: $T_1$ and $T_2$ are definitionally equivalent if each of the vocabulary

[14] Hodges (1993, p. 4) uses the term "signature" with a meaning identical to that of "vocabulary" as used by Ebbinghaus and Flum (and as used in this paper).

items for $T_1$ can be given an explicit definition (as a first-order formula) in terms of the vocabulary items for $T_2$ (and vice-versa). The explicit definitions are the basis for declaring that the theories are definitionally equivalent, and they also provide the blueprint of the process for turning any model of one theory into a definitionally equivalent model of the other theory. For a proof of the result and details of the related concepts and processes, see (Hodges 1993, pp. 58-62).

The present work will rely on a particularly simple special case of definitional equivalence: that where one predicate symbol is defined as equivalent to another (the two symbols must have the same arity). An example of such an explicit definition is

$$\forall x\big(P_a(x) \leftrightarrow Q_a(x)\big)$$

Given a theory $T_1$ with a vocabulary that contains $P_a$ but not $Q_a$, a definitionally equivalent theory $T_2$ can be constructed by replacing every occurrence of $P_a$ in $T_1$ with $Q_a$. $T_2$ then has a vocabulary that contains $Q_a$ but not $P_a$. For any model $M_1$ of $T_1$, if we alter the model by adding a relation for $Q_a$ with an extension identical to that of $P_a$ and then removing the relation for $P_a$, the resulting structure is guaranteed to be a model of $T_2$.

It bears emphasizing that definitional equivalence does not mean that theories $T_1$ and $T_2$ are identical; they are distinct theories. Nor does it mean that structures $M_1$ and $M_2$ are identical; they are distinct structures. $M_1$ and $M_2$ are definitionally **equivalent**. This is a consequence of the fact that first-order logic, both syntactically and semantically, is only concerned with relational properties; it does not attempt to interpret the internal syntax of the notations used for vocabulary symbols. If you relabel some of the predicates in an FOL theory, leaving everything else the same, you have said essentially the same thing using different terms.

### *9.5.2 Grammar Translation 1: $P_a \rightarrow Q_a$*

Let Σ be an alphabet, $\tau_1$ be the vocabulary $\{<\} \cup \{P_a | a \in \Sigma\}$, and $G_1$ be an FOL$[\tau_1]$ grammar. The stringset described by $G_1$ is

$$L_1 = strset(G_1)$$

Each string $w \in L_1$ is a model $M_1^w$ of the form

$$M_1^w = \left(D^{M_1^w}, <^{M_1^w}, \left(P_a^{M_1^w}\right)_{a\in\Sigma}\right)$$

We will translate grammar $G_1$ into an equivalent one which uses alphabet predicate symbols $Q_a$ in place of $P_a$. The purpose for this will become clearer in the next section.

Let $\pi_{ID}$ be the identity map on Σ: $(\forall a \in \Sigma)(\pi_{ID}(a) = a)$. Note that $\pi_{ID}$ is a permutation. We use $\pi_{ID}$ to define a bijective function $\rho_1$ for predicate symbols. $\rho_1$ simply maps each predicate symbol $P_a$ for alphabet symbol $a$ to a corresponding predicate symbol $Q_a$ for the same alphabet symbol $a$.

$$(\forall a \in \Sigma)\big(\rho_1(P_a) = Q_{\pi_{ID}(a)}\big)$$

Translate the vocabulary $\tau_1$ into the vocabulary $\tau_2$ by replacing each predicate symbol $P_a$ with the symbol $\rho_1(P_a) = Q_{\pi_{ID}(a)} = Q_a$. There are no symbols of the form $Q_a$ in $\tau_1$, so this is well-defined, and the resulting vocabulary is

$$\tau_2 = \{<\} \cup \{Q_a | a \in \Sigma\}$$

Translate grammar $G_1$ into the grammar $G_2$ (with $G_2$ defined on $\tau_2$) by replacing each occurrence of an alphabet predicate symbol $P_a$ with the predicate symbol $\rho_1(P_a) = Q_a$, leaving

all of the other symbols unaltered. $G_2$ is thus an FOL$[\tau_2]$ grammar (a sentence of FOL$[\tau_2]$). $G_1$ and $G_2$ are not the same grammar, but they are definitionally equivalent.

For any model $M_1^w$ of $G_1$, we can construct a corresponding model $M_2^w$ of $G_2$ by changing the labels on the relations of the model according to $\rho_1$:

$$M_2^w = \left(D^{M_2^w}, <^{M_2^w}, \left(Q_a^{M_2^w}\right)_{a\in\Sigma}\right)$$

where

$$D^{M_2^w} = D^{M_1^w}$$
$$<^{M_2^w} = <^{M_1^w}$$
$$(\forall a \in \Sigma)\left(Q_a^{M_2^w} = \rho_1^{-1}\left(Q_a^{M_2^w}\right) = P_{\pi_{ID}^{-1}(a)}^{M_1^w} = P_a^{M_1^w}\right)$$

In other words, predicate $Q_a$ has the same extension in model $M_2^w$ as predicate $P_a$ has in model $M_1^w$, for every $a \in \Sigma$. If we meta-interpret $Q_a(x)$ in the same way as $P_a(x)$, to mean that position $x$ has the property of terminal symbol $a$, then $M_2^w$ represents the very same string $w$ as $M_1^w$. The structures $M_1^w$ and $M_2^w$ are not the same structure, but they are definitionally equivalent, and they represent the same string $w$.

By extending the sense of the term, we could say that $L_1 = strset(G_1)$ and $L_2 = strset(G_2)$ are also definitionally equivalent, given that $L_1$ and $L_2$ are sets of models. If we focused instead on just the sequences of alphabet symbols represented by the models, then the sets of such "raw" strings are identical.

This first translation from $G_1$ to $G_2$ hasn't resulted in a different stringset in any meaningful sense. But it has given us a definitionally equivalent grammar, $G_2$, which does not contain any predicates of the form $P_a$. This makes it possible to cleanly perform the next translation, which will produce a definitionally equivalent grammar in which each $Q_a$ has been replaced with a symbol $P_b$, where $P_b \in \tau_1$, but it is possible that $a \neq b$.

### *9.5.3 Grammar Translation 2: $Q_a \rightarrow P_{\pi(a)}$*

Let $\pi$ be an alphabet permutation on Σ. We use $\pi$ to define a bijective function $\rho_2$ for predicate symbols. $\rho_2$ maps each predicate symbol $Q_a$ for alphabet symbol $a$ to a corresponding predicate symbol $P_{\pi(a)}$ for the alphabet symbol $\pi(a)$.

$$(\forall a \in \Sigma)\left(\rho_2(Q_a) = P_{\pi(a)}\right)$$

Translate the vocabulary $\tau_2$ into the vocabulary $\tau_3$ by replacing each predicate symbol $Q_a$ with the symbol $\rho_2(Q_a) = P_{\pi(a)}$. There are no symbols of the form $P_a$ in $\tau_2$, so this is well-defined, and the resulting vocabulary is

$$\tau_3 = \{<\} \cup \{P_a | a \in \Sigma\}$$

Note that, in fact, $\tau_3 = \tau_1$. They are the same set of symbols. This translation will be producing a different grammar than $G_1$, but defined on the same vocabulary.

Translate grammar $G_2$ into the grammar $G_3$ (with $G_3$ defined on $\tau_3$) by replacing each occurrence of an alphabet predicate symbol $Q_a$ with the predicate symbol $\rho_2(Q_a) = P_{\pi(a)}$, leaving all of the other symbols unaltered. $G_3$ is thus an FOL$[\tau_3]$ grammar (a sentence of FOL$[\tau_3]$). $G_2$ and $G_3$ are not the same grammar, but they are definitionally equivalent.

For any model $M_2^w$ of $G_2$, we can construct a corresponding model $M_3^v$ of $G_3$ by changing the labels on the relations of the model according to $\rho_2$:

$$M_3^v = \left(D^{M_3^v}, <^{M_3^v}, \left(P_a^{M_3^v}\right)_{a\in\Sigma}\right)$$

where

$$D^{M_3^v} = D^{M_2^w}$$
$$<^{M_3^v} = <^{M_2^w}$$
$$(\forall a \in \Sigma)\left(P_a^{M_3^v} = \rho_2^{-1}\left(P_a^{M_3^v}\right) = Q_{\pi^{-1}(a)}^{M_2^w}\right)$$

In other words, predicate $P_a$ has the same extension in model $M_3^v$ as predicate $Q_{\pi^{-1}(a)}$ has in model $M_2^w$, for every $a \in \Sigma$. The structures $M_2^w$ and $M_3^v$ are not the same structure, but they are definitionally equivalent. They do **not** necessarily represent the same string, however.

### *9.5.4 Composing the Translations: $P_a \to P_{\pi(a)}$*

Since $\tau_3 = \tau_1$, $G_3$ is an FOL$[\tau_1]$ grammar. However, the only way that $G_3 = G_1$ is if $\pi = \pi_{ID}$, so that $P_a$ replaced $Q_a$ for every $a \in \Sigma$. If $\pi$ is any other permutation, then some symbol $Q_a$ was replaced by $P_b$ where $a \neq b$.

If we think about the composition of the two translations, feeding the result of translation 1 into translation 2, the overall effect is to convert one FOL$[\tau_1]$ grammar, $G_1$, into another FOL$[\tau_1]$ grammar, $G_3$, by substituting each occurrence of $P_a$ in $G_1$ with $P_{\pi(a)}$. This is the FOL counterpart of the grammar transformation for an $SL_k$ grammar: the "roles" played by each of the alphabet symbols is shuffled according to the alphabet permutation $\pi$.

Recall that for $G_1$ the string $w$ is a model $M_1^w$ of the form

$$M_1^w = \left(D^{M_1^w}, <^{M_1^w}, \left(P_a^{M_1^w}\right)_{a\in\Sigma}\right)$$

The counterpart of $M_1^w$ for $G_3$ is $M_3^v$, which is of the form

$$M_3^v = \left(D^{M_3^v}, <^{M_3^v}, \left(P_a^{M_3^v}\right)_{a\in\Sigma}\right)$$

where

$$D^{M_3^v} = D^{M_1^w}$$
$$<^{M_3^v} = <^{M_1^w}$$
$$(\forall a \in \Sigma)\left(P_a^{M_3^v} = P_{\pi^{-1}(a)}^{M_1^w}\right)$$

$M_3^v$ represents the string $v$. The relation $P_a^{M_3^v}$ contains the positions where symbol $a$ appears in $v$. $P_a^{M_3^v}$ is the same set as $P_{\pi^{-1}(a)}^{M_1^w}$, which contains the positions where symbol $\pi^{-1}(a)$ appears in $w$. Said in the other direction, for each position in which $b \in \Sigma$ appears in $w$, $\pi(b) \in \Sigma$ appears in the same position in $v$.

Put a third way, $v = \beta(w)$, where $\beta(w)$ is the string substitution for $\pi$. This is the same substitution function as realized by the transformation of SL grammars: each symbol of $v$ is the result of applying the alphabet permutation $\pi$ to the corresponding symbol of $w$. The translation of $M_1^w$ to $M_3^v$ is the model-theoretic realization of $v = \beta(w)$.

### *9.5.5 Pulling it Together*

Recall the definition of the stringset transformation for $\beta$ of $L$, repeated here:

$$t_{lang}(L) = \{\beta(w) | w \in L\}$$

The same definition can be used for FOL$[\tau]$ grammars:

$$strset(G_3) = \{\beta(w) | w \in strset(G_1)\} = t_{lang}\big(strset(G_1)\big)$$

For any FOL$[\tau]$ grammar $G_x$ describing stringset $L_x$, There exists an FOL$[\tau]$ grammar $G_{x\pi}$ which describes the stringset $L_{x\pi} = t_{lang}(L_x)$ for the alphabet permutation $\pi$.

## *10. Final Conclusions*

### *10.1 Purely Computational Classes are Incapable of Accounting for Phonological Typology*

The narrow result of this paper is that the class of strictly local grammars wildly overgenerates stress patterns. The minimum value of *k* required to describe many attested stress patterns is already sufficient to allow many grammars that are not only unattested but qualitatively distinct from all attested stress systems. The class of strictly *k*-local grammars for any given value of *k*, on its own, cannot provide even a reasonable approximation to the typology of human stress systems.

This paper did not attempt anything close to a complete exploration of the typology predicted by the class of strictly local grammars; such a study was unnecessary to make the point. The derangement method shows that for each strictly local grammar describing an attested pattern, there are numerous other strictly local grammars, many of which generate unattested patterns. The derangement method makes this easy to see because of the proven result that any derangement of a strictly local grammar *G* generates the analogous derangement of the language of *G*. The space of strictly *k*-local grammars for any given value of *k* is finite, but finiteness is insufficient to prevent substantial overgeneration.

The wider result of this paper is that the same problem can be expected to plague the classes of the subregular hierarchy generally. For starters, many subregular classes of languages are a superset of the strictly local languages and inherit all of the overgenerations of the strictly local languages. More generally, NC consists of all of the subregular languages. The subregular languages are coextensive with the languages described by First-Order Logic sentences. The second proof, in Section 9, proves that all FOL grammars also have the derangement property: an appropriate derangement of the grammar (FOL sentence) describes the derangement of the corresponding language. We now argue that this is not due to any fluke property that happens to be common to SL and NC specifically, but is endemic to classes of languages that are defined purely in traditional computational terms.

The subregular classes can be largely characterized by the type of first-order logic sentences that are adequate to describe them (McNaughton & Papert 1971, Rogers & Pullum 2011). The strictly local languages can all be described by conjunctions of negative string literals (a negative string literal is what we have been labeling as constraints), which are equivalent to FOL sentences of a restricted form, as shown in section 9.4. The Locally Testable (LT) languages can all be described using propositional logic, which is equivalent to FOL sentences of a less restricted form. The NonCounting (NC) languages can all be described by sentences of FOL without further restriction apart from those enforcing the requirements of stringsets (see section 9.2). In general, the subregular classes are distinguished by what restrictions are imposed on the permitted FOL sentences[15].

[15] Such restrictions can include limiting the sentence to use of the precedence ordering relation and not the successor relation (an example is the class Strictly Piecewise), or limiting the sentence to use of the successor relation and not the precedence relation (an example is the class Strictly Local). Once the power of first-order logic is reached, the

The syntax of FOL distinguishes predicates only by their arity. Any predicate symbol can appear where any other same-arity predicate symbol can appear in well-formed formulae. An atomic formula is any predicate with an appropriate number of well-formed terms for its arguments, and non-atomic formulas are defined inductively in terms of formulas without further qualification (van Dalen 2013). This is not an accident or a convenience, but a deliberate part of the purely relational nature of the logic. In terms of logical structure, all of the derangements of an FOL sentence are "equivalent".

This property is intentionally reflected in the definitions of "computational" classes of languages: the classes are distinguished by the relational properties of the alphabet symbols across the strings of the language, and not by the use of any specific alphabet symbols or extrinsic interpretations placed on those symbols. Any computational class admitting a particular language must also include all of the partial derangements of that language.

Given the place of NC at the top of the subregular hierarchy, any language of any subregular class can be described by some sentence of FOL. For any class of FOL grammars based solely on the logical structure of the sentences, the inclusion of one logic sentence entails the inclusion of all of the partial derangements of that logic sentence. The fact that the computational classes of languages can be characterized by restrictions on the logical form of the FOL sentences describing those languages serves to amplify the fact that the inclusion of one language in a computational class entails the inclusion of all of the derangements of that language.

While it is possible to logically define classes using reference to specific predicate symbols, such classes would not be classically computational in nature. Computational classes are quite deliberately indifferent as to what terminal symbols are used in the alphabet, and in FOL grammars those symbols are referenced by predicate symbols.

With respect to phonology, this means that the subregular classes are defined without reference to any phonological substance (e.g., primary vs. secondary stress). Phonological substance only directly arises in the semantic interpretation of the predicate symbols (that is, the relations of the models) and thus could only be referenced in FOL grammars by selective reference to some predicate symbols apart from others, which computational class definitions do not do by design.

The derangement method serves to highlight precisely the symmetries that result from the inability of computational class definitions to selectively reference and restrict specific predicate symbols. Those symmetries are at odds with the numerous asymmetries of substance that are at the heart of the empirical issues discussed in this paper. Further, those symmetries are at odds with asymmetries of substance inherent to phonological typology: stress is generally attracted to heavy syllables not light syllables, stress does not "harmonize" across adjacent syllables in the way that some vocalic properties can, onsets and codas have different behaviors cross-linguistically, etc.

The conclusion here is that the massive overgeneration predicted by the existing subregular classes cannot be addressed by constructing additional subregular classes, or indeed any new computational classes, as they will suffer the same fate. Computational classes may or may not play some explanatory role in phonological typology, but they cannot be the entire explanation, or even most of the explanation.

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successor relation is derivable from the precedence relation, so FOL with the precedence relation subsumes FOL with the successor relation.

### *10.2 Prospects for Improving Restrictiveness*

One natural way to approach this would be to express the additional, substance-laden restrictions as additional FOL sentences that are required to be part of all FOL grammars, in the same way as the conditions that must be satisfied in order for structures to qualify as strings at all.

We are currently unaware of any detailed proposals that restrict phonological grammars to some subregular class or union of subregular classes while supplementing them with additional restrictions that are not computational in nature, and using the combination as a theory of phonological typology. We have nothing concrete along those lines to evaluate at the current time. We can, however, offer some observations that may be relevant to such efforts.

In order for additional FOL sentences (or their equivalents) to actually restrict the typology, those sentences would need to be present in all grammars. If the additional restrictions were optional, then all of the typologically problematic grammars would still be present, supplemented by further grammars including one or more of the additional restrictions. In other words, the additional substance-based restrictions would need to be universals.

One other approach to analyzing stress phenomena that is similar in spirit is work on modeling the formal languages as relations rather than as stringsets. Instead of a set of stressed strings, which could be labeled outputs, the relational approach defines a language as a set of input string / output string pairs, where the output string bears the final stress pattern assigned to the corresponding input. Koser (2022) has taken this approach, using finite-state transducers as the descriptive formalism for the relations. We have not (yet) examined transducer-based accounts of stress, but we do not see any a priori reason why they would be less subject to the inherent symmetries of computationally-based classes. We expect that deranging the description of a transducer-based stress analysis would yield effects similar to the deranging of subregular grammars for stringsets.

### *10.3 Final Word*

The derangement method highlights precisely the symmetries that result from the inability *of computational classes* to selectively reference specific symbols. Those symmetries do not match the numerous asymmetries characteristic of stress (and phonology). Pure computational theory is concerned only with relational properties of stringsets; it is not concerned with the semantic interpretation of the symbols and symbol sequences. Phonology, in contrast, is quite concerned with the interpretations of the symbols; a stressed syllable has a different status than an unstressed syllable *cross-linguistically*. An adequate theory of phonology needs to account for the asymmetries characteristic of attested phonological systems.

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